Math 220 - Student Interviews for Study Methods and Proof Problem-Solving (think-aloud)

March & April 2011

Name:
Student #:
Term and Section of Math 220:
Describe:

- Purpose of interview.

- Outline of the interview.

- Consent form and payment + signature

Part A: Demographic and Study Skills Questions

Demographic:

1. What program and major are you in?

2. What year of your program are you in?

3. Why are you taking/did you take MATH 220?

4. Did you enjoy MATH 220?

5. What MATH couses did you take in Fall 2010? What MATH courses are you taking this term?

6. What are your plans after university? What career do you hope to pursue?

Study Methods:

1. Please rank each of the following items (1 = least, 8 = most) for the **amount of time** you spent on them while studying for your MATH 220 exams:

   ( ) Read chapter(s) in the textbook.

   ( ) Took notes on chapter(s).

   ( ) Read class notes.

   ( ) Transcribed class notes into personal notes.

   ( ) Read solutions to the homework problems or midterms.

   ( ) Redid problems from the homework problems or midterms.

   ( ) Did additional problems from the textbook.

   ( ) Reworked examples from the textbook and/or class notes.

2. Please rank each of the following items (1 = least, 8 = most) for how **useful** you think they are for studying for MATH 220 exams:

   ( ) Read chapter(s) in the textbook.

   ( ) Took notes on chapter(s).

   ( ) Read class notes.

   ( ) Transcribed class notes into personal notes.

   ( ) Read solutions to the homework problems or midterms.

   ( ) Redid problems from the homework problems or midterms.

   ( ) Did additional problems from the textbook.

   ( ) Reworked examples from the textbook and/or class notes.

3. When you did practice problems for MATH 220:

   (a) How did you choose which problems to do?

   (b) How did you decide when you have done enough?

4. Did you study with friends? Why or why not?.

5. Did you go to office hours? Why or why not?

6. Did you go to the drop-in tutoring centre? Why or why not?

7. Did you work with a private tutor? Why or why not?

8. Did you use the Learning Goals for the course? If so, how? If not, why not?

9. Did you do anything else to study for MATH 220? If so, what?

Part B: Warm-up Think Aloud Problem

Prove the following statement:

"If x is rational and y is rational, then $x + y$ is rational."

Part C: Think Aloud Problem 1

Prove or disprove each of the following statements for arbitrary sets A, B, and C.

(a) $(A \cup B) - C \subseteq (A - (B \cup C)) \cup (B - (A \cap C))$

(b) $(A - (B \cup C)) \cup (B - (A \cap C)) \subseteq (A \cup B) - C$

Possible Hints:

1. What do you need to decide first?

2. Can you do anything to help you choose whether to prove or disprove these statements?

3. Can you draw a visual representation of these sets?

For (a) only:

1. The statement is true.

2. Is there a usual method for showing that one set is a subset of another?

3. Start by assuming that a is an element of the left hand side and show that it must also be an element of the right hand side.

For (b) only:

1. The statement is false.

2. In general, how can you show that a statement is false?

3. Would looking at specific examples be helpful?

4. Can you use a diagram to help you develop a counterexample?

Part D: Think Aloud Problem 2

Assume that $g : A \to B$, $h : A \to B$, $f : B \to C$ and $f \circ g = f \circ h$.

(a) Prove or disprove that if f is surjective then $g = h$.

(b) Prove or disprove that if f is injective then $g = h$.

Possible Hints:

For (a):

1. How do you choose whether to prove or disprove?

2. Can you try an example?

3. Can you draw a visual representation of the statement?

4. What does it mean for f to be surjective?

5. The definition of f being surjective is:
   "A function $f : B \to C$ is surjective if and only if for every y in the codomain C there is at least one x in the domain B such that $f(x) = y$."

6. What does it mean for $g = h$?

7. $g = h$ is equivalent to $g(x) = h(x) \ \forall x \in A$.

8. What does it mean for $f \circ g = f \circ h$?

9. $f \circ g = f \circ h$ is equivalent to $f \circ g(x) = f \circ h(x) \ \forall x \in A$.

10. The statement is false.

11. How can you show that a statement is false?

12. You need to give a counterexample.

13. What do you need in this case to have a counterexample?

14. You must define sets A, B, C, and functions f, g, and h such that $f : B \to C$, $g : A \to B$, $h : A \to B$ where $f \circ g = f \circ h$ but $g \neq h$.

15. It can be helpful to choose small sets A,B,C and to define f,g and h explicitly on these sets.

For (b):

1. How do you choose whether to prove or disprove?

2. Can you try an example?

3. Can you draw a visual representation of the statement?

4. What does it mean for f to be injective?

5. The definition of f being injective is:
   "The function $f : B \to C$ is injective if for all a and b in B, if $f(a) = f(b)$, then $a = b$. Equivalently, if $a \neq b$, then $f(a) \neq f(b)$."

6. What does it mean for $g = h$?

7. $g = h$ is equivalent to $g(x) = h(x)$ $\forall x \in A$.

8. What does it mean for $f \circ g = f \circ h$?

9. $f \circ g = f \circ h$ is equivalent to $f \circ g(x) = f \circ h(x)$ $\forall x \in A$.

10. The statement is true.

11. What methods of proof do you know of?

12. Could you use proof by contrapositive?

13. What is the contrapositive of the statement?

14. The contrapositive of the statement is "if $g \neq h$ then f is not injective."

15. What do you need to assume to prove the contrapositive?

16. You start by assuming that there exists $a \in A$ such that $g(a) \neq h(a)$.

17. What do you need to show/conclude to prove the contrapositive?

18. You need to show that there exists $b_1, b_2 \in B$ such that $b_1 \neq b_2$ but $f(b_1) = f(b_2)$.