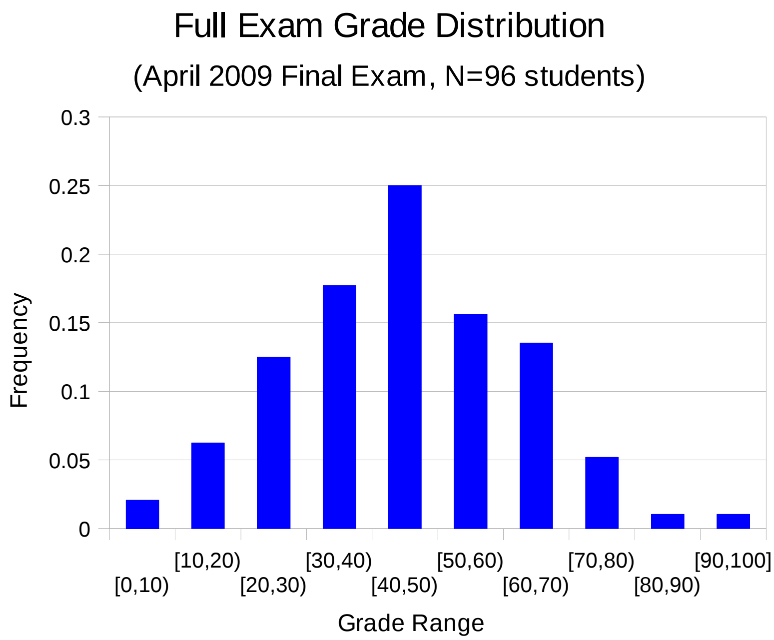
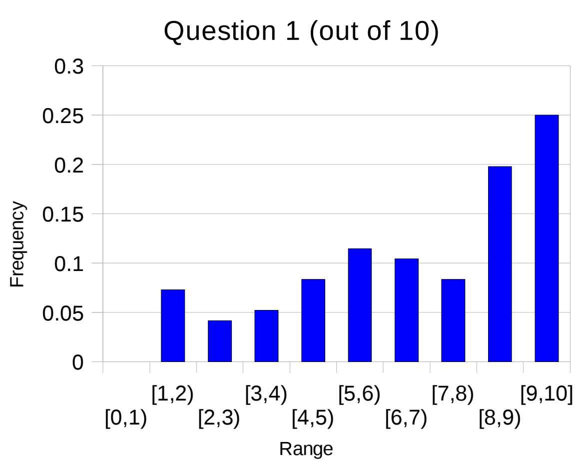
MATH 220 Final Exam – April 2009
Summary of Error Coding

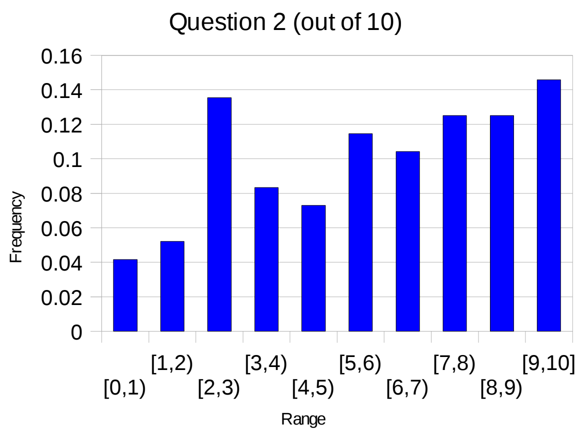
Summary Statistics for the Full Exam:
N=96 students
Mean: 44.56
Median: 43.5
Histogram:



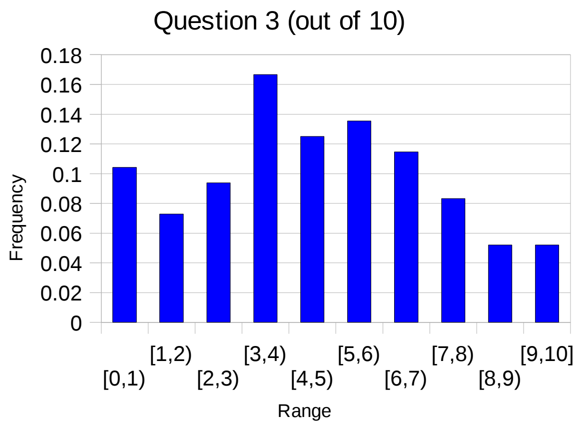
Grade Breakdown by Question:
Q1. Please give precise mathematical definitions of the following: (see below for individual part breakdown)
Mean: 6.38/10
Median: 7/10
Histogram:



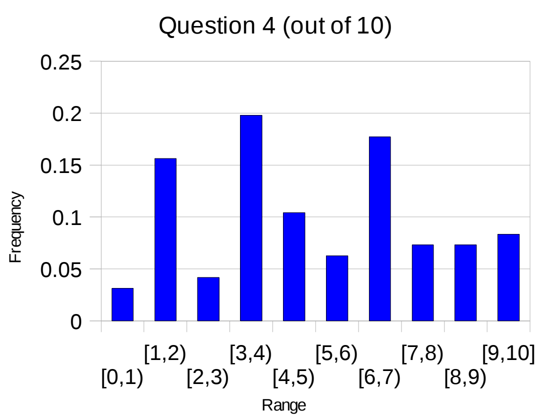
Q2. Functions (3 parts)
Mean: 5.35/10
Median: 5.5/10
Histogram:



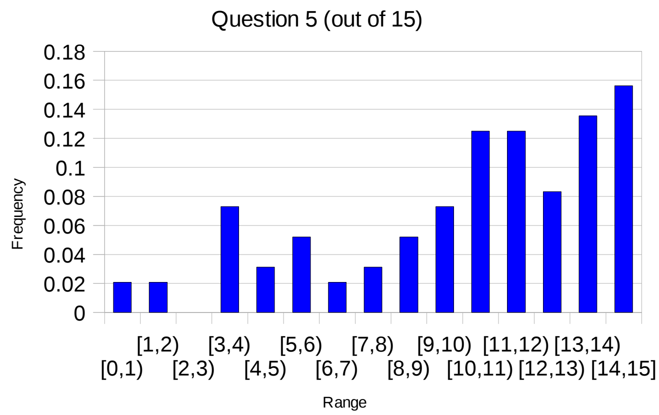
Q3. Logic and short proofs (4 parts)
Mean: 4.13/10
Median: 4/10
Histogram:



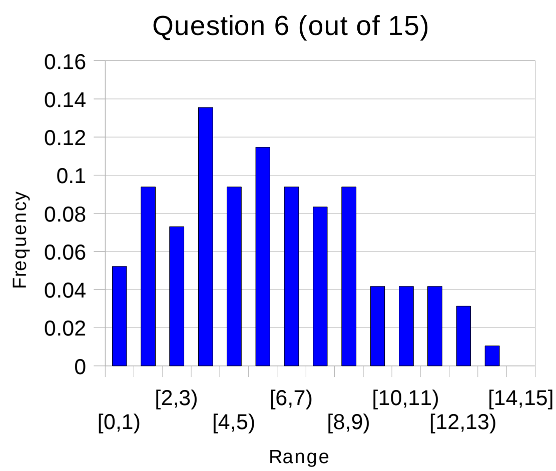
Q4. Proofs about functions and cardinality (2 parts)
Mean: 4.54/10
Median: 4/10
Histogram:



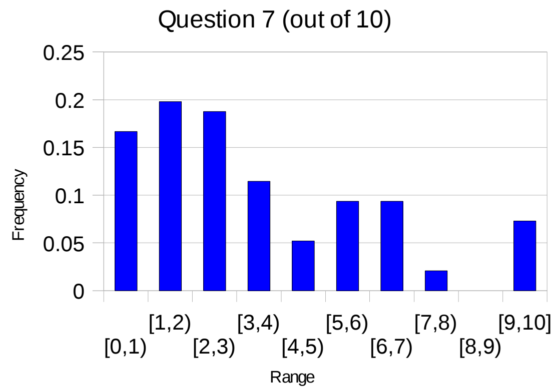
Q5. Induction Proofs (3 parts)
Mean: 9.67/15 (64%)
Median: 10.5/15
Histogram:



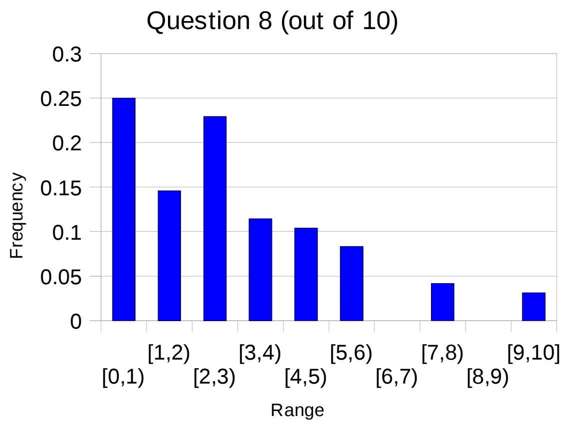
Q6. Sequences and series proofs (4 parts)
Mean: 5.25/15 (35%)
Median: 5/15
Histogram:



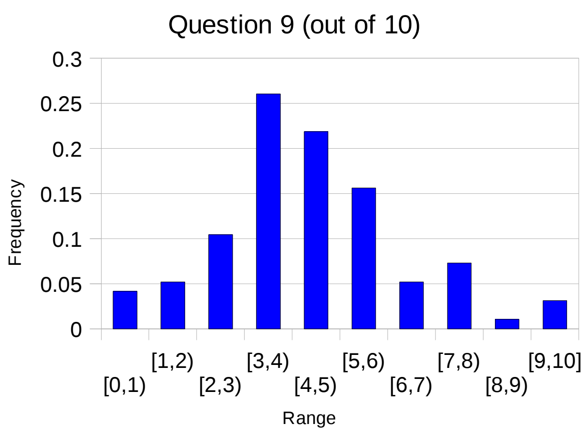
Q7. Proofs applying sequence convergence (2 parts)
Mean: 2.98/10
Median: 2/10
Histogram:



Q8. Proofs/disproofs applying sequence convergence (2 parts)
Mean: 2.38/10
Median: 2/10
Histogram:



Q9. Sup/Inf and Max/Min (2 parts)
Mean: 3.9/10
Median: 4/10
Histogram:



<u>Coding of Specific Questions:</u>

Question 1 – Definitions: (means are listed in parentheses)

(a) surjective function (83%)
(b) an uncountable set (78%)
(c) the well-ordering principle of N (76%)
(d) the principle of mathematical induction (48%)
(e) an indexed intersection of sets and an indexed union of sets (52%)
(f) the infimum of a set of real numbers (63%)
(g) the preimage/inverse image of a set (60%)
(h) what it means for a sequence to converge (71%)
(i) what it means for a sequence to diverge to infinity (60%)
(j) what it means for an infinite series to converge (46%)

Question 4 – Proofs about functions and cardinality:

(a) Prove that the given function is bijective (8 marks)

Mean:3.21/8

3 students did not attempt this problem
1 student clearly did not know the definition of bijective
the remaining students (92) attempted to prove injective and surjective

Injective:

- 24 produced a correct direct proof
- 4 produced a correct proof by contradiction
- 54 attempted to produce a direct proof by failed to complete it or had errors
- 3 attempted to prove the contrapositive (and failed)
- 5 clearly did not know or misused the definition of injective
- 22 failed to simplify sufficiently (and gave up)
- 26 skipped important steps or missed cases (ex. The case ab=-1)
- 20 made algebra errors
- 7 assumed the conclusion
- 1 “proved by example”

Surjective:

- 16 produced a correct direct proof (by construction)
- 38 attempted to directly prove, but failed or had errors
- 12 clearly did not know or misused the definition of surjective
- 17 had algebra errors, or couldn't solve for the variable
- 7 skipped important steps (like verifying that their choice works)
- 6 didn't simplify enough (and gave up)
- 4 chose the wrong root, or used both roots
- 5 assumed the conclusion
- 5 wrote it as a computation, not as a proof
- 2 thought that the sum of two surjective functions is surjective
- 1 “proved by example”

(b) Prove that the cardinality of the half-line is equal to that of the whole real line (2 marks)

Mean: 1.38/2

15 students did not attempt this part (or did not do sufficient for me to consider it an attempt)
59 students correctly answered, by invoking the bijection from part (a)
5 answered correctly, but did not explicitly invoke the bijection from (a) (and so lost marks)
1 answered correctly, by constructing a new bijection
2 indicated that they knew they needed to find a bijection, but were unable to do so
5 constructed new bogus “bijections” (all proved incorrectly that their function was bijective)
1 used “proof by verbal argument”
7 used faulty facts and terminology about cardinalities of sets
(ex. “N and R have the same cardinality”, “both are infinite so they have the same cardinality” “all uncountable sets have the same cardinality”, “all countable/denumerable sets have the same cardinality”)

Question 6 – Proofs about sequences and series (from first principles):

(a) Simplify the nth partial sum of a given series (involving logarithms) (3 marks)

Mean: 1.23/3

13 did not attempt the problem or did not do sufficient for me to consider it an attempt
5 (possibly more) thought they were asked to prove the convergence or something
11 only looked at the infinite series (did not seem to know what the nth partial sum was)
4 only simplified the nth term, not the nth partial sum
3 translated the sequence or series notation incorrectly
14 used a bogus log rule (ex. log(a+b)=log(a)+log(b))
8 made algebra errors (other than log rule errors)
6 didn't think to make a common denominator
8 didn't think to use the * or / rule for logarithms
2 nearly completed it, but didn't think to cancel the terms
2 did not simplify log(1)

*many students left their answer in terms of k, rather than n, but did not lose marks for this.

(b) Prove from first principles that a given sequence converges to 1 (4 marks)

Mean: 1.35/4

8 did not attempt the problem or did not do sufficient for me to consider it an attempt
3 did not appear to know the definition of convergence, or misused it
15 made algebra errors
35 made erroneous inferences about inequalities (I have a list of types of errors)
7 made errors using absolute values (like more, but hard to differentiate from inequality errors)
10 either didn't complete the chain of inequalities, or developed a useless chain (ex. End with 3)
6 were unable to “invert” to get N (indicated that they needed to, or tried but could not)
2 just took N=1/eps with no justification
4 applied limit laws
3 had “sloppy” proofs (missing steps?)

*nearly all students seemed to know what they were expected to do for this problem (ie. Approached it as a direct proof using the definition of convergence, and tried to develop a chain of inequalities.)
*there were massive numbers of errors involving inequalities and absolute values

(c) Prove from first principles that a given sequence converges to b. (3 marks)

Mean: 1.68/3

12 did not attempt the problem or did not do sufficient for me to consider it an attempt
58 assumed c>0 either in their choice of N or in their inequality chain
4 made algebra errors
4 made erroneous inferences about inequalities
2 were unable to “invert” to get N
2 assumed the conclusion
2 applied limit laws
1 just took N=1/eps with no justification
1 tried to use induction

4 wrote it as a computation, not a proof

*nearly all students seemed to know what they were expected to do for this problem (ie. Approached it as a direct proof using the definition of convergence, and tried to develop a chain of inequalities.)

(d) Prove that a given infinite series converges and find its limit (using part c) (5 marks)
Mean: 1.03/5

29 did not attempt the problem or did not do sufficient for me to consider it an attempt
13 couldn't think of how to break up the terms or performed a very flawed partial frac decomp.
9 made algebra errors in their partial frac. decomp.
7 made other algebra errors
2 made erroneous inferences about inequalities
11 only looked at convergence of the sequence (not of the partial sums)
5 failed to invoke part c, or failed to justify using it
2 applied the ratio test
1 applied the comparison test
2 applied limit laws
1 assumed the conclusion

*many of the students received 0 because they only computed the first few terms of the series (no other work)

Question 8 – Apply the (known) convergence of sequences to prove/disprove other results:
(a) Prove that if two sequences converge to different values, then there is some index beyond which the two sequences' terms are never equal. (7 marks)
Mean: 1.8/7

22 did not attempt the problem or did not do sufficient for me to consider it an attempt
40 attempted to prove directly, 2 were successful
16 attempted to prove by contradiction, 1 was successful
4 attempted to prove the contrapositive, none were successful
5 attempted to "prove by example"
4 attempted to "prove by verbal argument"
5 did not do enough to identify their method of proof
14 either did not recall, incorrectly recalled, or misused the definition of convergence
14 did not use a common epsilon for the two sequences
9 did not use a common N for the two sequences
21 failed to choose a useful common epsilon to use in the convergence definition
13 applied an incorrect inequality "rule" (ex. Triangle inequality with -)
18 made incorrect inference about inequalities
6 skipped important steps/justifications in their proof
2 attempted to used the relationship between a and b in problem 7

*I did not see any drawings/graphs indicating attempts to visualize the situation... did they do this on scrap paper, or not at all?

(b) Prove or disprove that if two sequences converge to the same value then there is some index at which the sequence terms are equal (3 marks)

Mean: 0.6/3

44 did not attempt the problem or did not do sufficient for me to consider it an attempt
15 provided valid counterexamples
5 provided invalid counterexamples
18 attempted to directly prove (incorrectly) that it was true
5 attempted to prove by contradiction (incorrectly) that it was true
14 made erroneous inferences about inequalities (mostly those trying to directly prove it)
5 applied an incorrect inequality "rule"
5 missed important steps or cases (usually in invalid counterexamples)
5 misused the definition of convergence or its notation
2 used "proof by example"

*A few students (2 or 3) used graphs to illustrate that their sequences did not cross, but did converge to the same value
*Students chose to "prove" or "disprove" in roughly equal numbers. Do they have any ideas for how to get a "feel" for this before they attempt to prove/disprove?