MATH 110 – Differential Calculus

Learning Goals

Course-level learning goals:

In this course students will learn the basic ideas, tools and techniques of differential calculus and will use them to solve problems from real-life applications. The topics covered in this course can be separated into three major types: precalculus, differential calculus and applications.

A: Precalculus:

Students will learn the basic skills in mathematics which prepare them for learning the calculus topics. Specifically, students will learn

A1. to evaluate, simplify and manipulate *basic functions* which includes

- polynomials,
- radical functions,
- trigonometric functions,
- inverse trigonometric functions,
- exponential functions,
- logarithmic functions,
- absolute-valued functions,
- functions that are constructed by additions, subtractions, multiplications, divisions, exponentiations and/or compositions of the above functions,
- piecewise functions;

A2. to find the domain, range and intercepts of a basic function (see A1), and the behaviour of such a function at/near the endpoints of the domain;

A3. to solve linear, quadratic, rational, radical, trigonometric, exponential, logarithmic, and absolute-valued equations;

A4. to solve linear, quadratic, rational, radical, trigonometric, exponential and logarithmic inequalities;

A5. to construct new functions by applying function composition and identify the various functions that make up a composite function;

A6. to relate graphs to simple functions such as linear, quadratic, power, root, reciprocal, absolute-valued, trigonometric, inverse trigonometric, exponential, logarithmic and piecewise functions as well as equations involving circles and ellipses; i.e., plot a graph from a given equation and find the equation from a given graph;

A7. to find intersections of two or more graphs;

A8. to find a value of a function from its graph and determine whether a point of given coordinates lies on the graph;

A9. to find the distance between two given points and the slope/equation of the line containing two given points;

A10. to translate, scale and reflect graphs;

A11. to construct a graph from a function that is the reciprocal or the inverse of another function;

A12. to construct the reciprocal or the inverse graph from another graph;

A13. to construct a graph from a given context and extract information related to a given context from a graph;

A14. to apply the Pythagorean theorem, write down trigonometric relationships involving the sides and angles of a right triangle, and express proportional relations between similar triangles;

A15. to compute the area of basic 2D shapes, and the surface area and the volume of basic 3D shapes;

A16. to construct neat, logical, understandable explanations and solutions to problems involving multiple steps.

B: Differential calculus:

Students will learn the basic ideas, tools and techniques of differential calculus which prepares them for solving application problems. Specifically, students will learn

B1. the idea of limit and to evaluate limits involving basic functions (see A1) using the limit laws, the squeeze theorem and/or the l'Hospital's rule;

B2. the relationship between limits and asymptotes and to find asymptotes using limits;

B3. the idea of continuity and to construct/determine functions that are continuous or discontinuous using the definition of continuity or theorems involving continuity;

B4. the idea of derivative in terms of the slope of the tangent line to a curve, the rate of change of a quantity with respect to another quantity, and the limit definition of derivative;

B5. the idea of differentiability and to construct/determine functions that are differentiable or non-differentiable;

B6. the graphical/numerical relationship between a function and its derivative;

B7. to differentiate basic functions (see A1) using the definition of derivative or the differentiation rules (derivative formulas, product rule, quotient rule, chain rule, logarithmic differentiation);

B8. to implicitly differentiate an equation involving two variables and to find tangent lines to the graph of an implicit function;

B9. to find the critical points and the local/absolute maxima/minima of a function defined on any open/closed interval;

B10. to find the intervals of increase/decrease of a function using derivative tests;

B11. to find the intervals of concavity, and to find the inflection points of a function defined on any interval using derivative tests;

B12. to graph a function by analyzing the behaviour of the function using limits and derivatives;

C: Applications

Students will apply the above skills and knowledge (both precalculus and calculus) to translate a problem involving higher-level abstractions or real-life applications into mathematical problems and solve. In general, when solving a problem students should be able to

- after reading a problem, correctly state in their own words what the problem is asking and what information is given that is needed in order to solve the problem;
- after restating the problem, identify which mathematical techniques and concepts are needed to find the solution;
- apply those techniques and concepts and correctly perform the necessary steps to obtain a solution;
- interpret results within the problem context and determine if they are reasonable.

Specifically, students will learn

C1. the idea of linear approximation and to find linear approximations to functions;

C2. the idea of approximation error and to estimate the error bound of a linear approximation;

C3. to solve application problems involving velocity and acceleration of moving objects, rate of change, economics, natural growth/decay, related rates, linear approximation and optimization by using basic mathematics, limits, and derivatives.

Students will also learn how to construct simple proofs. They will show that a given mathematical statement is either true or false by constructing a logical argument using appropriate explanations, theorems and properties of functions. Specifically, students will learn

C4. to properly read a theorem or an implication and construct the contrapositive statement;

C5. to prove (or disprove) a statement or a mathematical formula by logical arguments without using the same statement or formula being proved;

C6. to apply a theorem by satisfying its hypotheses and drawing logical conclusion, or by negating the conclusion and concluding that not all the hypotheses are satisfied;

C7. the intermediate value theorem, Rolle's theorem and mean value theorem, and to use these theorem to prove mathematical statements.

Workshop-level learning goals:

Workshops are integral parts of the course so students are required to attend every workshop. The main goals of the workshops are

W1. to practice problem-solving techniques which apply to homework/test problems;

W2. to enhance mathematical knowledge and develop communication skills in math by writing solutions, explaining/defending own work/ideas, evaluating and criticizing peers' work;

W3. to learn how to cooperate/interact with peers and solve new problems using the knowledge obtained in class;

W4. to receive feedback from peers and TAs which help identify and correct possible mistakes/weaknesses/misconceptions.