Teaching methods comparison in a large introductory calculus class

a place of mind
THE UNIVERSITY OF BRITISH COLUMBIA

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Introduction

Our work is motivated by a demand for empirical study of less-traditional but evidence-based instructional methods for introductory calculus at the undergraduate level (Speer, Smith III, & Horvath, 2010). We gleaned structural ideas from the Physics Education Research community, especially the recent study of Deslauriers, Schelew, & Wieman (2011) involving a week-long intervention by an alternate instructor, though instructional decisions in our study were based on research on student learning in mathematics, with an attempt to situate our analysis in the Action Process Object Schema (APOS) framework. At the research-focused institution in question, the calculus course under study has a population with a range of backgrounds consisting primarily of Commerce and Economics students. Though some applications are tied to these disciplines, much of the syllabus is in common with Calculus 1 elsewhere in North America. Traditional lecture remains the (nearly) uniform choice for instruction, though some local pressure to examine teaching methods has recently arisen.

Our research questions are not unlike those of Deslauriers et al. (2011), though we have enhanced the experimental design to improve validity:

Q1: Compared to more traditional lecture-based instruction, will students demonstrate more sophisticated reasoning on an immediate test of learning when high-engagement instruction is implemented for a single topic (100-150 minutes of class time)?

Q2: Will any effects persist to later, standard tests of learning in the course?

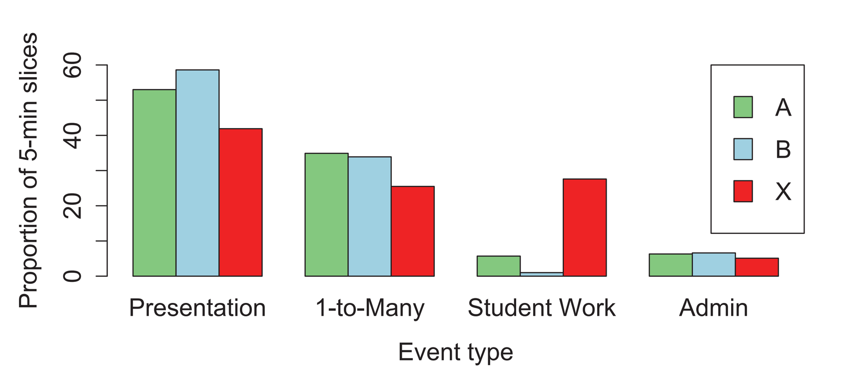
Instructional methods

Lesson structures borrowed ideas from Peer Instruction (Crouch & Mazur, 2001), the Activities, Classes, Exercises (ACE) cycle from the Mathematics Eduation Research community (Weller et al., 2003) and general principles about learning that are now available (Bransford, Brown, & Cocking, 2003) but are not known to many university mathematics faculty, particularly at research-focused institutions. Specifically, the goal was to promote "active learning" as described in the science education literature; much of the evidence arises from the K-12 setting though there has been some study at the post-secondary level (Hake, 1998; Michael, 2006).

Standard week: **Lecture with questions**
- Chalkboard lecture
- Clicker questions (1-2/hour)
- Whole-class discussions led by instructor

"Intervention week": **Higher engagement**
- Pre-class assignment

In class:
- Structured handout
- More clicker questions (5-8/hour)
- Small group tasks



Classroom activity quantified by the Teaching Dimensions Observation Protocol (Hora & Ferrare, 2009) based on events during 5-min slices of lecture period; classified as Presentation, meaning lecture with no student interaction, 1-to-Many interaction which includes student questions and whole-class discussion led by the instructor, Student Work involving peer discussion and/or desk work, or Admin, non-content administrative events.

Experimental Design

1st Term, 1st Year Course where 95% of students have previously taken calculus.

Two comparable (by diagnostics) sections, 150 and 200 students, taught by instructors that are good examples of lecture-based classroom.

Junior instructor trained in research-based methods takes over for one topic (100-150 minutes of in-class time) in each section.

Course weeks →

Section A									
A_1	A_2	$A_3 \ldots$	A_7	$\mathbf{X_8}$	A_9	A_{10}	A_{11}	A_{12}	
Section B									
B_1	B_2	$B_3 \ldots$	B_7	B_8	B_9	B_{10}	$\mathbf{X_{11}}$	B_{12}	
Assessments in common									
att	D			Q_{RR}	M_{RR}		Q_{LA}	att	FE

See righthand column for description of these items.

Assessments

- Diagnostic items (locally developed): **att**: Math Attitudes and Perceptions Survey similar to CLASS survey for Physics (Adams et al., 2006) and **D**: Precalcus and Calculus diagnostic tests 15-20 minutes in length. Distributions on all of these were similar for the two sections.
- Quizzes at the very end of a topic's instructional period, $\mathbf{Q_{RR}}$ and $\mathbf{Q_{LA}}$, based on agreed-upon learning goals but written separately from all instructional materials. Results below.
- Midterm test contained a common related rates question ($\mathbf{MT_{RR}}$); final exam (**FE**) had common questions on both topics.

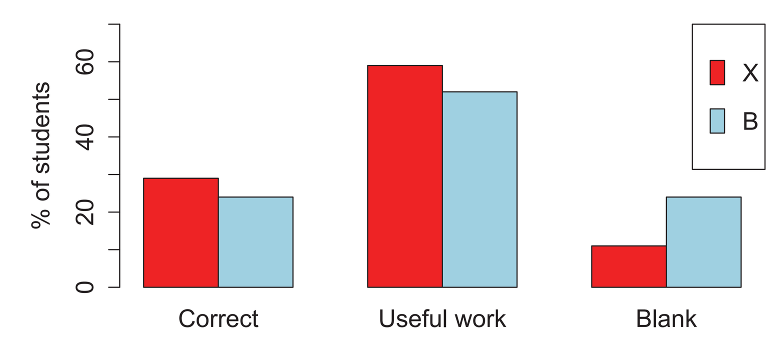
Results and discussion

All p-values below refer to a Chi-squared test comparing populations for the associated item; those represented by the graphs refer to proportions in mutually exclusive categories.

Related Rates Quiz (Q_{RR})

For a growing conical pile of gravel with height equal to base diameter, draw and label a diagram then determine the rate of change of height in time given the volume increase rate and initial size.

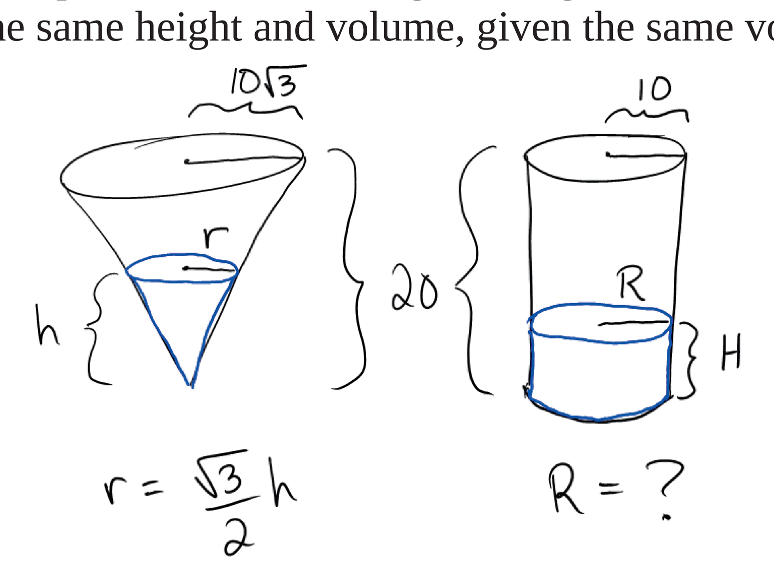
In Section X (N=177), more students made progress in producing a sensibly-labeled diagram (75% versus 60%, p<0.02) and in solving the problem (graph at right, p < 0.02). Unlike Section X, Section A (N=131) had seen a complete worked example during class with the "cone geometry" for an inverted conical tank draining, and were more likely to use the standard proportionality relationship for the radius and height, but were still not as successful in their solutions.

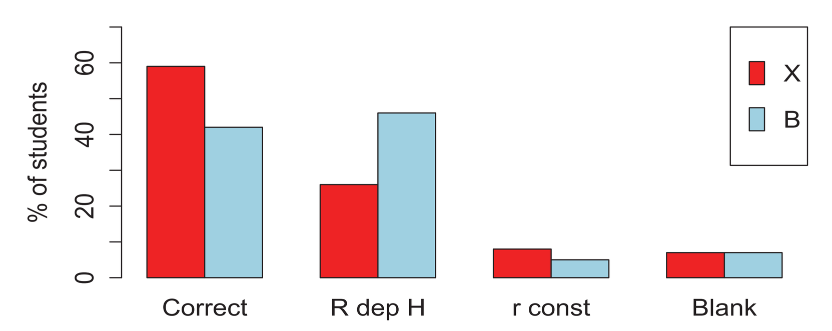


Solution progress in gravel pile quiz problem.

Related Rates Midterm Question (MT_{RR})

Compute the rate of change of height for each of an inverted cone and cylinder-shaped water tank of the same height and volume, given the same volume fill rate and initial water depth.



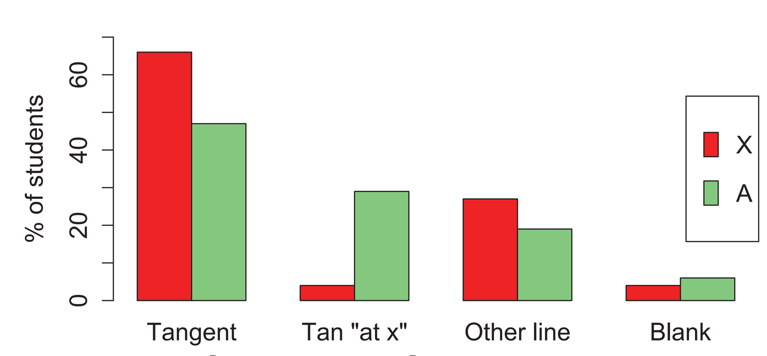


Radius relation for paired tanks midterm problem.

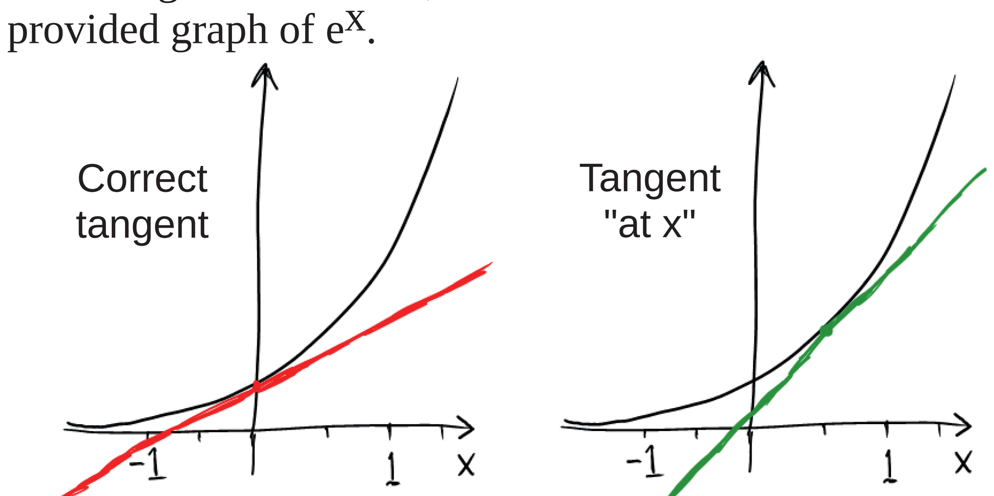
Many more students in Section B (N=127) applied the proportional relation formula for the radius and height of the cone directly to the cylinder ("R dep H", meaning they computed R to be a function of H instead of constant). Students treating both tank radii as constant ("r const") were a small minority. Section X (N=174) students were more likely to treat the radii correctly (p < 0.01).

Linear Approximation Quiz (Q_{LA})

Estimate $e^{0.5}$ using linear approximation with the tangent line at x=0, determine if it is an over- or underestimate, and draw the situation on the provided graph of e^x.



Drawing the tangent in linear approximation.



Here, N=106 for Section X and N=133 for Section A. Successful use of the linear approxoimation formula $f(x) \sim f(a) + f'(a)(x-a)$ was similar in both sections, with 52% in Section X and 60% in Section A computing the estimate correctly (p > 0.2). Students in Section X were less likely to declare their estimate an underestimate (66% versus 82% for Section A, p < 0.01), but were much more successful in drawing the correct tangent line; 29% of the Section A students drew the linearization at 0.5 (diagram on right, the Tan "at x" error) instead of at 0 (on left) despite having seen multiple correct diagrams presented during class.

Final Exam (FE)

Performance on the final exam is still under review, but the "gap" in working with tank radius appears a bit smaller on the final exam, while analysis of the error in linear approximation favors the higher-engagement section (computational items were still similar between sections).

Main conclusions

We observed better performance on conceptual components of the related rates assessments, and a much larger number of students were able to demonstrate the correct picture for linear approximation; the higher-engagement section was stronger in each case. Performance in both sections was very close on computational items and concepts more strongly tied to earlier parts of the course. The data from the final exam was only somewhat supportive of our second research question; some of the gap in performance on the radius in the tank problems persisted, and students in the higher-engagement section were more likely to connect the second derivative to an error bound in linear approximation.

References

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Further information:

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Look for our preliminary report in the Proceedings of the 15th Annual Conference on Research in Undergraduate Mathematics Education (to be released 2012).