Learning Outcomes

MATH 104/184

Differential Calculus with Applications to Commerce and Social Sciences

PRELUDE: We expect students to have a reasonable mastery of the Principles of Mathematics 12 curriculum. No formal review of this material occurs in MATH 104/184.

Below are the major detailed learning outcomes for MATH 104/184. Not all of these outcomes are of equal importance. Clearly some of these may be included in others; they are stated as extra guidance for your study.

1. You should be able to understand the idea of *derivative*: you should be able to explain what a derivative is in terms of the idea of a tangent line to the graph of a function, how a derivative can be used to describe the rate of change of one quantity with respect to another, and how to relate the geometric ideas to the analytic ideas.

2. You should be able to give an intuitive explanation of the process of taking a *limit*. You should be able to compute basic limits of functions.

3. You should be able to explain the relation between limits and the process of differentiation. You should be able to state the definition of derivative as the limit of the difference quotient and compute the derivative of a simple function directly using limits.

4. You should be able to explain the notion of continuity as related to functions. You should be able to state and apply the mathematical definition of continuity, and compare and contrast it to the high school notion of a continuous function as one whose graph you can draw without lifting your pen.

5. You should be able to compare and contrast the ideas of continuity and differentiability.

6. You should be able to explain the relationship between the derivative and linear approximation. You should be able to compute the linear approximation to a function at a given point, to use this approximant to estimate values of the function, and to analyse the error involved in doing this.

7. You should understand and be able to apply the rules of differentiation: constant-multiple rule, sum rule, product rule, quotient rule, chain rule. You should be competent at computing the derivatives of given functions; this implies you should know the derivatives of the elementary functions.

8. You should be able to explain the relationship between the derivative of a function as a function and the notion of the derivative as the slope of the tangent line to a function at a point.

9. You should be able to use derivatives to explore the behaviour of a given function. You should be able to apply the first and second derivative tests and interpret their results in order to locate and classify the extreme points and inflection points of the function. You should be able to use this information to describe and sketch the graph of the function.

10. You should be able to recognize and explain what an implicitly defined function is. You should be able to apply implicit differentiation to find linear approximations to implicit functions.

11. You should be able to solve related rates problems.

12. You should be able to interpret the idea of optimization as the procedure used to make a system or a design as effective or functional as possible, and translate it into a mathematical procedure involving finding the maximum/minimum of a function. You should be able to solve extreme-value problems and discuss the importance of boundary conditions in these problems. You should also be able to relate the mathematical solution to such a problem to the real-world.

13. You should understand exponential and logarithm functions: you should be able to graph these functions and do computations involving these functions, including finding their derivatives. You should be able to solve problems involving exponential growth and decay, and involving compound interest and future and present value of money.

14. You should be able to apply calculus to solve basic problems involving concepts from economics such as marginal cost, marginal revenue, marginal profit, and elasticity of demand.

15. You should understand the basic trigonometric functions and their inverses: you should be able to graph these functions and do computations involving these functions, including finding their derivatives. You should also be able to explain and to use the idea of branches of the inverse trigonometric functions.

16. You should be able to compute a Taylor polynomial of a specified degree about a given point for a given function. You should be able to use basic Taylor series for computations.

17. You should be able to implement Newtons method to compute the zeros of a function. You should be able to explain how Newtons method works.

    Remark: In the end, you should aim for a level of understanding that (a) allows you to carry out computations with ease, (b) allows you to apply your technical skills to actual problems, and (c) allows you to write an essay entitled "What is Calculus?".