MATH 110 Midterm 1, October 15th, 2014
Duration: 90 minutes
*This test has **7 questions** on **9 pages**, for a total of 50 points.*

- Read all the questions carefully before starting to work.
- All questions require a full solution; you should give complete arguments and explanations for all your calculations; answers without justifications will not be marked.
- Continue on the back of the previous page if you run out of space.
- Attempt to answer all questions for partial credit.
- This is a closed-book examination. **None of the following are allowed**: documents, cheat sheets or electronic devices of any kind (including calculators, cell phones, etc.)

First Name: ______________________ Last Name: ______________________

Student-No: ______________________ Section: ______________________

Signature: ____

Question:	1	2	3	4	5	6	7	Total
Points:	8	7	10	6	6	7	6	50
Score:								

Full-Solution Problems. In the following questions, justify your answers and **show all your work**. Unless otherwise indicated, **simplification of answers are required**.

2 marks

1. (a) Here is a table of values for two functions $f(t)$ and $g(t)$

t	-2	-1	0	1	2	3	4
$f(t)$	4	-1	2	8	1	0	5
$g(t)$	-2	-1	1	1	3	2	0

Compute the following values.
(*i*) $f(g(1))$
(*ii*) $g(f(0))$
(*iii*) $f(f(-2))$

3 marks

(b) Find the domain of the following function

$$h(x) = \sqrt{2x+5} + \frac{x-3}{x^3 - 5x^2}.$$

3 marks

(c) Consider the parabola $y = -x^2 - x + 6$. Let A be the point of intersection of the parabola with the positive x-axis and let B be the point with positive x-coordinate obtained from the intersection of the parabola with the line $y = 4$. Find the distance between the points A and B.

7 marks

2. Determine whether each of the following limits exists, and find their value if they exist.

(a) $\lim_{x \to 2} \frac{1}{2}x^3 - 3$.

(b) $\lim_{x \to -3^+} \sqrt{x+3}$.

(c) $\lim_{x \to 1} f(x)$, where $f(x) = \begin{cases} 3 & \text{if } x \leq 1 \\ x+3 & \text{if } x > 1. \end{cases}$

(d) $\lim_{x \to 0} \frac{4x^2 - 2x}{x^2 + 3x}$.

(e) Suppose $\lim_{x \to 0}(f(x) + g(x)) = 1$ and $\lim_{x \to 0}(f(x) - g(x)) = 2$. Find $\lim_{x \to 0} f(x)g(x)$.

3. A rocket is launched from a space station. The rocket's position (measured in metres) at time t (measured in seconds) is given by

$$p(t) = t^2 + t + 1, \quad t \geq 0.$$

2 marks

(a) Find the average velocity of the rocket over the interval from $t = 1$ to $t = 3$ seconds. Include units in your answer.

2 marks

(b) Let $h \neq 0$ be a small real number. Find the slope of the secant line between the points $(1, p(1))$ and $(1 + h, p(1 + h))$. Simplify your answer.

2 marks

(c) Using limits, explain how you can calculate the exact value of the instantaneous velocity of the rocket at $t = 1$, and calculate this value (include units). No marks will be given if the velocity is calculated using methods that do not involve limits.

1 mark

(d) Find the equation of the tangent line to the graph of $p(t)$ at $(1, p(1))$.

2 marks

(e) Sketch the graph of the position function $p(t)$. On the same sketch, draw the tangent line found in part (d).

1 mark

(f) Now think about the function $v(t)$ representing the rocket's velocity at time t. Which of the following options best describes $v(t)$? Circle your answer.

A. Decreasing and positive

B. Decreasing and negative

C. Increasing and positive

D. Increasing and negative

E. Constant

$\boxed{\text{6 marks}}$ 4. Let a, b be real numbers and put

$$f(x) = \begin{cases} -1 & \text{if } x \leq -1 \\ ax^3 + b & \text{if } -1 < x \leq 1 \\ \dfrac{x^2 - 1}{x - 1} & \text{if } x > 1. \end{cases}$$

For what values of a and b is f continuous on $(-\infty, \infty)$? Make sure you show all your work and justify your claims.

$\boxed{\text{5 marks}}$ 5. (a) Using the theorem(s) we discussed in class, carefully prove that $x^4 - x^3 + 2x^2 - 1 = 0$ has a solution between $x = -1$ and $x = 1$. Make sure you justify your claims.

$\boxed{\text{1 mark}}$ (b) Explain why the equation in part (a) has at least another solution between $x = -1$ and $x = 1$.

7 marks 6. Sketch the graph of a function f which satisfies all of the following criteria:

- The domain of f is $[-3, 1) \cup (1, 3]$
- $\lim_{x \to -1^-} f(x) = 0$
- $\lim_{x \to -1^+} f(x) = 1$
- $\lim_{x \to 1} f(x) = 3$
- The tangent line to the graph of f is horizontal at $x = 2$
- The tangent line to the graph of f has a negative slope at $x = -2$.

Make sure you label the axes and clearly identify the points on the graph that are related to the above conditions. *Note:* You are NOT required to provide a formula for $f(x)$.

7. Let

$$f(x) = \begin{cases} x^2 - 4 & \text{if } x \leq 2 \\ |x - 5| & \text{if } x > 2. \end{cases}$$

4 marks

(a) Sketch the graph of $f(x)$.

2 marks

(b) Consider $f(x)$ given above, and let

$$g(x) = \begin{cases} -1 & \text{if } x \leq 2 \\ 2 & \text{if } x > 2. \end{cases}$$

Where is $f(x) - g(x)$ continuous? Explain why.