Mathematics 446 — Spring 2005 — eighth assignment

This is due next Wednesday, March 10.

1. I have tried and not really succeeded in class to prove geometrically that the golden ratio is not a rational ratio. Complete this proof, giving details. Use Euclid's Lemma that an interval halved successively is smaller than any given interval. You might want to look also at the handout from Euclid.

2. Find the continued fraction expansions of $\sqrt{7}$, $\sqrt{11}$, $\sqrt{13}$.

3. Prove that if x has a periodic fraction expansion then it is a root of a quadratic equation with integral coefficients.

4. Describe all solutions of $x^2 - Ny^2 = 1$ for $N = 11$ by the Indian method. Explain as you go. Also $N = 23$, $N = 71$.

5. Finish the proof started in class that
$$p_{n+1} = q_{n+1}p_n + p_{n-1}$$
$$r_{n+1} = q_{n+1}r_n + r_{n-1}$$

Use mathematical induction.