

## Mathematics 446 — sixth assignment — due Wednesday, November 26

There are some problems here involving difficult computation. There are many ways to do them, but keep in mind that you might be expected to do similar questions on an exam.

**Exercise 1.** Use the binomial theorem to calculate  $\sqrt[3]{999}$  correct to 17 decimals after the decimal point.

**Exercise 2.** Assuming you were using the series

$$\int_0^x \frac{dt}{1+t^2} = \arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} \pm \dots$$

with  $x = 1/\sqrt{3}$ , how many terms would you have to use to get  $\pi$  to 20 decimals?

**Exercise 3.** Use the Taylor series for  $e$  to compute it to 17 decimals' accuracy.

**Exercise 4.** Read Dedekind's essay. (a) [deleted] (b) Prove that if  $x_n$  is an increasing sequence of numbers that is bounded, then it converges to a number  $x$ . (c) Prove that if  $x_n$  is a sequence of terms which alternate in sign and decrease in value, then the series

$$x_1 + x_2 + \dots + x_n + \dots$$

converges to a real number.

**Exercise 5.** (a) Prove that the sequence

$$x_n = (1 + 1/n)^n$$

is increasing and bounded and therefore converges to some real number. (b) [deleted]

**Exercise 6.** (a) Define the product of two real numbers in Dedekind's sense. (b) Using Dedekind's definitions, prove that  $\sqrt{2}\sqrt{3} = \sqrt{6}$ .