Topic: Laser beam shaping in industrial applications.

(Lighthearted introduction about Archimedes, Reagan and Fresnel lenses omitted.)

Light emitted by a laser is monochromatic, unidirectional and of coherent phase. These characteristics facilitate a wide variety of industrial applications such as microphotlithography, materials processing, laser writing and medicine. In some applications, it is desirable that a laser emits a beam with a particular intensity distribution and shape. An optical system is used to shape the beam into the desired form.

Optical systems may consist of a simple aperture mask (a lossy system), diffractive optics, or refractive optics. The last type is the most relevant to this course and the easiest to understand.

We will deal with the simple example of converting a round beam with Gaussian intensity distribution into one with a flattop intensity distribution.
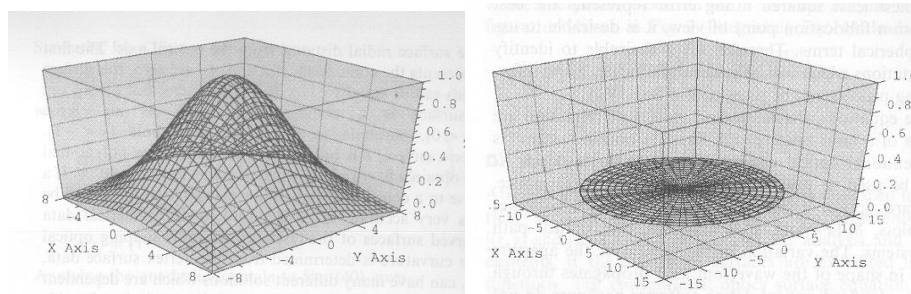
[images from Dickey & Holswade 2000]

Since a primary characteristic of laser light is its phase coherence, our optical system must preserve this. In other words, the shape of the beam wave-front must remain unchanged.

Consider the following two-lens system:
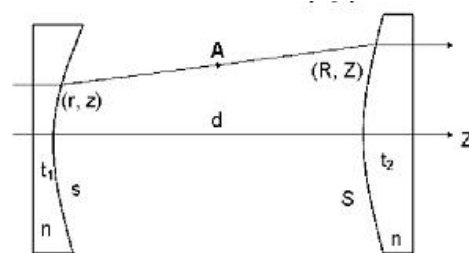
The lenses are separated by distance d at the inner surfaces of their axes, and they have index of refraction n. Excluding the curved portion, the lenses have thickness t_1 and t_2.

A ray of light entering the system horizontally from the left is refracted by surface s at the point (r,z), taking on the direction of the vector A, and is refracted by the surface S at the point (R,Z), exiting the system horizontally again at the right.

The system must produce a beam with the following characteristics:
- Uniform intensity distribution.
- Uniform optical distance for all paths taken.

Let's deal with the first condition. We can think of intensity (i.e. energy) being conserved throughout a bundle of rays. In calculus terms, this means:

$$I_{in}\,dw = I_{out}\,dW$$

Over the entire cross-section of the input and output beams, we express this as an integral:

$$\int_0^{2\pi} d\theta \int_0^r I_{in}(r)r\,dr = \int_0^{2\pi} d\theta \int_0^R I_{out}(R)R\,dR$$

For the input beam, we express intensity with the following Gaussian function (over two dimensions):

$$I_{in}(r) = \exp[-2(r/r_0)^2]$$

Where r_0 is the waist of the beam – denoting the radius from the axis of the beam within which 86.5% of its energy is contained. Using this to solve the previous integral yields the fields the following expression for R as a function of r:

$$R = \sqrt{\frac{r_0^2}{2I_{out}}\left[1 - \exp\left(-2r^2/r_0^2\right)\right]}$$

…where:

$$I_{out} = \frac{r_0^2}{2R_{max}^2}\left[1 - \exp\left(-2r_{max}^2/r_0^2\right)\right]$$

Now for the second condition:
The optical path length of a ray passing through the axis of the system is:

$$(OPL)_0 = nt_1 + d + nt_2$$

A ray passing the system at distance r from the axis will have the following optical length:

$$(OPL)_r = nz + [(R-r)^2 + (Z-z)^2]^{1/2} + n(t_1 + d + t_2 - Z)$$

Since wave-front uniformity must be preserved optical distances must be equal, i.e. $(OPL)_0 = (OPL)_r$ Substituting the previous equations yields:

$$\left[(R-r)^2 + (Z-z)^2\right]^{1/2} = n(Z-z) - d(n-1)$$

…which may be re-written to solve for $(Z-z)$ as a function of r:

$$(Z-z) = \frac{n(n-1)d + \left[(n-1)^2 d^2 + (n^2-1)(R-r)^2\right]^{1/2}}{n^2 - 1}$$

Note that R has already been expressed as a function of r. If rays are refracted at the surfaces s and S according to Snell's Law, then the ray-tracing equation for the system is:

$$(R\text{-}r)(\mathbf{A})_z = (Z-z)(\mathbf{A})_r$$

Where A is the following ray vector:

$$\mathbf{A} = \frac{z'\left[n - \sqrt{1 + z'^2(1-n^2)}\right]}{1 + z'^2}\mathbf{r} + \frac{\left[nz'^2 + \sqrt{1 + z'^2(1-n^2)}\right]}{1 + z'^2}\mathbf{k}$$

Solving for z' yields:

$$z' = \frac{-(R-r)(Z-z) \pm n\sqrt{(Z-z)^2 + (R-r)^2}}{\left[1 - n^2\right](Z-z)^2 - n^2(R-r)^2}$$

Note that R and $(Z-z)$ have both been expressed in terms of r. Integrating z' with respect to r allows us to find values for z and Z. Thus the optical system may be specified according with data points for (r,z) and (R,Z).

In practice, analytical integration of z' is very difficult. The Riemann Sum method allows for a reasonable approximation – which is the approach that I have taken in my simulation of results.

Once data points for (r,z) and (R,Z) have been obtained, one would typically use a least-squares method to fit the lens system to simple functions that are spherical or conical, since these forms are the most economical to manufacture with minimal error.

For an example of real-world output from a laser emitter using an optical system as described here, please view the following brochure:

http://www.vlt.nl/documents/PDF/Lasers/snf.pdf

References:

1. Dickey & Holswade. 2000. *Laser Beam Shaping: Theory and Techniques*. Chapter 4. ISBN-082470398-7.