

Mathematics 309 — Spring 2004 — Fourth homework

Due Wednesday, February 25.

1. Find the principal planes and focal length of a sphere of radius R .
2. Consider a single convex (bulging out on both sides) lens, radius R_1 on one side, R_2 on the other, d in between. Find its principal planes and focal length.
3. If you place a square on the bottom of a still pool and look at it from above the surface of the water, does it always look like a square? This problem is not trivial. You might begin by realizing that with binocular vision, every point you see is actually a point or a virtual point situated in space. So the figure you see when you look at the square is in fact some figure in space, not only a flat diagram. Start with an easier question: does a straight line at the bottom of the pool look straight? Explain. Include some carefully done examples.
4. In class I “proved” that a magnifying lens always has the image upside down. But if you place two lenses in succession, each focussing on where the previous one forms its image, the image will be doubly reversed. Where does my “proof” fit in? Write down my proof and then explain what goes wrong in this case. Give a numerical example. Draw some pictures to explain things clearly.
5. Place two thin lenses of power P_1 and P_2 in sequence along the optical axis, so the left focal plane of the second is the same as the right focal plane of the first, you will get a telescopic lens system. Find a formula for the magnification of the system. Include some diagrams to explain what’s going on.
6. Consider a hemi-spherical lens of radius 1. Find its focal planes. Find the plane conjugate to the plane 1 unit to the to the left of the left focal plane. Find the linear image of a white spot on that plane, centered at $y = 0.5$, width 0.1 on the conjugate plane. Then find an approximate graph of the real image. (I’ll explain more about this in class. You should trace about 25 rays in all departing from the spot.)
7. Find out the indices of refraction of red and violet light in water. Calculate using calculus the angles at which the red and violet bands of primary and secondary rainbows appear in the sky.