Mathematics 256—Fall 1997

First homework—due Wednesday September 10

From the Introduction:

1. *At time* $t = 200$*, the object dropped from* $10,000$ km *has approximately*

$$\begin{aligned} r &= \quad 9920.66\,\text{km} \\ v &= -0.7995\ \text{km/}\textit{sec} \end{aligned}$$

Tell approximately what its height and velocity are at time $t = 201$.

2. *The differential equation in the introduction is rather special, since energy is conserved as the object falls. The kinetic energy is* $mv^2/2$ *and the potential energy is* $-mk/r$ *(a negative coefficient because the object loses potential energy as it falls). Recall that potential energy is defined up to some fixed constant, and here that constant is chosen to make the potential energy vanish at* $r = \infty$*. (a) Write down the expression for total energy. Solve it for* v *to obtain a first order differential equation for* r*. (b) Use the second order differential equation we derived above for* r *to show that energy is in fact constant.*

From Chapter 1:

3. *A cup of coffee initially at* $90°$ *sitting in a room at* $20°$ *takes* 5 *minutes to cool to* $70°$*. (a) What is the coffee temperature after* 10 *minutes? (b) What is the relaxation time? (c) How long does it take to cool to* $40°$*? (d) Write down a formula for the temperature at time* t *(given in minutes).*

4. *A radioactive substance emits radiation and changes to a new substance. The amount of radioactivity is proportional to the quantity of the original substance remaining. The half-life of a radioactive substance is the amount of time it takes for half the substance to decay radioactively. If the half life of radium is 1760 years, how much radium is left from an initial gram of radium after 100 years?*

5. *What is the exact relationship between half-life and relaxation time?*