Mathematics 103 — Section 202 — Spring 2001
Fifth homework — due Monday, February 26

Exercise 1. *(a) Find the following indefinite integrals using the substitution rule:*

$$(i) \quad \int \sin(3x)\,dx$$

$$(ii) \quad \int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$$

$$(iii) \quad \int x^3\sqrt{x^4+1}\,dx$$

$$(iv) \quad \int \frac{4}{1+2x}\,dx\ .$$

(b) Find the following definite integrals using the substitution rule:

$$(i) \quad \int_0^5 \sqrt{3+2x}\,dx$$

$$(ii) \quad \int_0^{\pi/4} \sin(4t)\,dt$$

$$(iii) \quad \int_0^4 \frac{dx}{(x-2)^3}$$

$$(iv) \quad \int_0^{\sqrt{\pi}} x\cos(x^2)\,dx\ .$$

Exercise 2. *Evaluate the following integrals. You must show all your work to get credit.*

(a) $\displaystyle\int \frac{1}{1-y}\,dy$

(b) $\displaystyle\int_0^T te^{-2t}\,dt$

(c) $\displaystyle\int \frac{2}{4+x^2}\,dx$

(d) $\displaystyle\int_2^p \frac{1}{1-y^2}\,dy$

(e) $\displaystyle\int_1^p \frac{1}{2+2y+y^2}\,dy$

(f) $\displaystyle\int_0^\pi x\sin\left(\frac{x}{2}\right)\,dx$

(g) $\displaystyle\int_1^S \frac{k_1}{k_2-n}\,dn$ $\quad$ *(k_2 outside the range $[1,S]$)*

Exercise 3. *Find the mean value of the function*

$$f(x) = \sin(\pi x/2)$$

over the interval $[0,2]$.

Exercise 4. *The intensity of light cast by a streetlamp at a distance x (in meters) along the street from the base of the lamp is found to be approximately $I(x) = 20^2 - x^2$ in arbitrary units for $-20 < x < 20$. (a) Find the mean intensity of the light over the interval $-5 < x < 5$. (b) Find the mean intensity over $-7 < x < 7$. (c) Find the value of b such that the mean intensity over $[-b, b]$ is $I_{\text{av}} = 10$.*

Exercise 5. *In November 1999, the rain in Vancouver fell at the rate $R(t) = 4\big((1 + t\sin(\pi t/30)\big)$ where t is time in days and $R(t)$ is in cm/day. Find the total amount of rain that fell and the mean rate of rainfall over the first 10 days of the month ($0 \le t \le 10$) and over the whole month ($0 < t < 30$).*

Exercise 6. *Consider a distribution function $y = f(x) > 0$ defined on some interval $[a, b]$. The* **median** *of f is defined to be a value of the independent variable, x, say $x = m$ which splits the area under $f(x)$ into two equal portions, i.e. such that*

$$\int_a^m f(x)\,dx = \int_m^b f(x)\,dx = \frac{1}{2}\int_a^b f(x)\,dx$$

Use this definition to find the median of the following functions on the indicated interval.

(a) $f(x) = 1 - x^2, \quad (-1 \le x \le 1)$

(b) $f(x) = |1 - x|, \quad (-1 \le x \le 1)$

(c) $f(x) = 5 - x, \quad (0 \le x \le 5)$

(d) $f(x) = \sin(2x), \quad (0 \le x \le \pi/4)$

[Remark: it will help to sketch the given function and interval and use considerations of symmetry for some of these examples.]