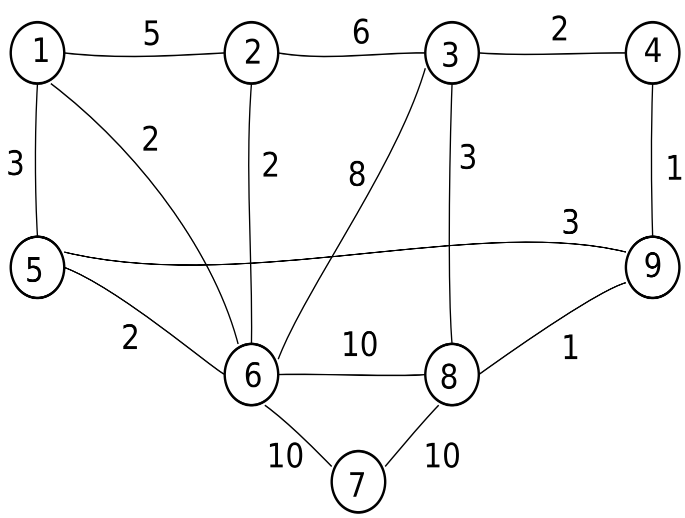
1. The (minimum) weight matching problem given in the text can be solved using the associated minimum cost flow problem. The problem in the text concerned the complete graph on 8 vertices with edge weights given below (they appear as a symmetric matrix). I have listed the dual variables $\pi_{i'}$ and $\pi_{i''}$ for the minimum cost flow problem. Use this to determine a solution for the fractional matching problem. Determine the edges which have equality labelling and then by finding a minimum matching on this subgraph you will obtain the optimal matching. (I mentioned in class that this example in the text was not helpful because at optimality $\gamma_{S_k} = 0$ for all odd sets S_k.)

$\pi_{i'} \backslash \pi_{i''}$	20	10	8	8	16	12	23	25
0		19	8	8	18	18	25	29
−8	19		0	8	10	4	15	23
−12	8	0		4	8	2	15	18
−14	8	8	4		2	10	15	16
−6	18	10	8	2		10	22	25
−6	18	4	2	10	10		19	19
5	25	15	15	15	22	19		37
7	29	23	16	16	25	19	37	

2. Consider a weighted matching problem which at optimality has no non zero γ variables. Can you argue in general that this implies there is no need for the primal/dual algorithm but merely one run of a maximum cardinality matching problem in such a case.

3. Solve the following maximum weight matching problem using our primal-dual algorithm.



4. Prove the following results concerning Maximum Weight Spanning Trees of a connected graph $G = (V, E)$ with edge weights $w(e)$. These ideas are useful in algorithms.

a) If $w(i, j) = \max\{w(i, k) : (i, k) \in E\}$ then there is a maximum weight spanning tree containing the edge (i, j).

b) If C is a cycle of edges $e_1, e_2, \ldots, e_t$ and $w(e_l) = \min_{1 \le i \le t} w(e_i)$ then there is a maximum weight spanning tree not containing the edge e_l.

c) (From a talk of B. Chazelle) Let $U \subseteq V$ have the property that for every pair of edges $e_1 = (i, j), e_2 = (k, l)$ where $j, k \in U, i, l \notin U$ there is a path P from j to k of edges, entirely in U, so that the minimum edge weight in the path P is at least $\min\{w(e_1), w(e_2)\}$. Then there is a maximum weight spanning tree of G which contains edges yielding a spanning tree on the vertices U.