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Imperfect price information and valuation by comparable sales

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Summary

Traditional valuation by comparable sales does not possess a formal methodology for the explicit incorporation of imperfect information. Chance-constrained programming permits this and, at its simplest, facilitates valuation by linear programming. All information, whether perfect or not, is incorporated into the methodology in a transparent manner. Together these features render the approach particularly useful where market information is otherwise scarce. The argument is illustrated by a practical example.

Keywords: valuation, imperfect information, chance-constrained programming.

1. Introduction

Traditional valuation by comparable sales developed historically to meet particular technical requirements. It relates a valuation to property characteristics; *ceteris paribus* the greater the amount of a particular (advantageous) characteristic the higher the value of the property as a whole. Further, the valuation is based on known realized sales prices of comparable properties. These facts are used in a context where, overall, there is very limited information; specifically, where only a few comparable sales are available. If instead of there being four or five comparable sales there were 12 to 15 and preferably twice as many again, an econometric-based valuation could be carried out (see Wiltshaw 1991a and 1991b) and Matysiak (1991) for, amongst other matters, a discussion of the applicability of econometrics to valuation).

Though traditional valuation by comparable sales works with limited information, it is assumed that information is accurate. Realized sales prices are considered to be exactly as stated. The characteristics of comparables are also treated as though they are measured precisely. This is assumed whatever the particular measurement scale used: a continuous characteristic, such as gross external area, is considered to be as accurately measured as a

Note: This paper was submitted in ignorance of Matysiak's (1992: *JPR*, 9(2), 114 21) reply to Wiltshaw's earlier article (1991: *JPR*, 8(2), 123 32) and does not constitute a reply to it.

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binary characteristic (like the possession, or not, of a south-facing aspect). Similar remarks apply to the property to be valued.

In summary, in traditional valuation it is assumed that, while there are only a few comparables, the property-specific information related to then can be relied upon completely. An *ad hoc* procedure is then followed to arrive at a valuation. However, this method has been challenged (Wiltshaw, 1991a) as imprecise, ambiguous and tautological. In the same paper it was suggested that linear programming represents a superior procedure. Nevertheless it is still the case that even this approach assumes all the data it uses is perfectly accurate.

The question now arises as to how we carry out a valuation in the face of imperfect information. In this particular paper we will confine our attention to imperfections in our knowledge of realized sales prices; in principle, however, other imperfect data can be incorporated into the valuation. Further, since the traditional method is fatally flawed, we will further limit the discussion to the linear-programming approach. The paper commences with a summary of the latter. This is followed by a formal statement of the imperfect information which is to be utilized. Section 4 presents the explicit incorporation of imperfect information into the mathematical programming valuation problem. Section 5 addresses the practical aspects of deriving linear algebraic constraints for comparables with imperfect price information. A simple example is then presented to illustrate the procedure. In the conclusions we debate briefly the potential contribution of the suggested methodology. Throughout the analytical focus continues to be a valuation consistent with market prices (Wiltshaw, 1991a), which may now be estimates rather than an explicit simulation of supply and demand conditions. The latter would require far more information than assumed here.

2. A summary of the linear programming approach

Valuers conceive of the value of a property as the total summation of the value of its characteristics. Let k_{ij} be the amount of property characteristic j to be found in property i; for example, in the case of a house, it may be the gross external area, the presence or absence of central heating, etc. Thus we can express the value of property i as:

$$\sum_{j=1}^{n} p_j k_{ij} \tag{1}$$

where p_j is the price of characteristic j and there are assumed to be n characteristics. The valuation problem can be interpreted as the discovery of the maximum price that can be attributed to the subject property, v which is algebraically and arithmetically consistent with the stated comparable sales. The analytical task is to input the characteristic prices ($p_1, p_2, \dots p_i \dots p_n$) from the information contained in the m comparable sales. It has been expressed (Wiltshaw, 1991a, p. 11) as:

$$\text{maximize: } \sum_{j=1}^{n} p_j k_{vj} \tag{2}$$

$$\left.\begin{aligned} \text{subject to: } & \sum_{j=1}^{n} p_j k_{1j} = c_1 \\ & \sum_{j=1}^{n} p_j k_{2j} = c_2 \\ & \quad \cdots \\ & \quad \cdots \\ & \quad \cdots \\ & \sum_{j=1}^{n} p_j k_{mj} = c_m \\ & p_1, p_2 \dots p_n \geq 0 \end{aligned}\right\} \tag{3}$$

where $c_1, c_2 \dots c_m$ are the m realized sales prices. The valuation is arrived at by inputing the characteristic prices $p_1, p_2 \dots p_n$ by means of the simplex algorithm.

3. Imperfect price information

If we consider the reality of valuation practice all the data required are unlikely to be equally available. We require information on the characteristics of both the subject property ($k_{v1}, k_{v2} \dots k_{vn}$) and the comparables, of which we will assume there are $m+t$ ($k_{11}, k_{12} \dots k_{1n}; k_{21}, k_{22} \dots k_{2n}; \dots k_{m1}, k_{m2} \dots k_{mn}; k_{m+1\,1}\; k_{m+1\,2} \dots k_{m+1\,n}; \dots k_{m+t\,1}, k_{m+t\,2} \dots k_{m+t\,n}$). This information, in its entirety, is assumed to be available to the valuer. However the latter may face two particular difficulties. All the realized prices of the comparables may not be known. In England and Wales, for example, property prices are treated as confidential information. This problem can be compounded by a thin market in traded properties, yielding, at best, few potential comparables. In what follows, where a comparable's realized prices is unknown it will be indicated by an asterisk; for example, in the case of comparable $m+i$ we write c^*_{m+i}.

Conceptually, at least, we are now in a position to argue that not only should the maximum value of subject property v be consistent with the m equality constraints for which the realized prices are known but, additionally, it should also be consistent with the equality constraints representing the comparables whose realized prices are not known to the valuer. We will assume there are t unknown prices. Algebraically the valuation problem can now be represented as:

$$\text{maximize: } \sum_{j=1}^{n} p_j k_{vj} \tag{4}$$

$$\text{subject to: } \left.\begin{array}{l} \sum_{j=1}^{n} p_j k_{1j} = c_1 \\ \sum_{j=1}^{n} p_j k_{2j} = c_2 \\ \cdots \\ \sum_{j=1}^{n} p_j k_{mj} = c_m \\ \sum_{j=1}^{n} p_j k_{m+1\,j} = c^*_{m+1} \\ \cdots \\ \sum_{j=1}^{n} p_j k_{m+t\,j} = c^*_{m+t} \\ p_1, p_2 \ldots p_n \geqslant 0 \end{array}\right\} \quad (5)$$

It is to be understood that for this to be a true linear programming problem $n > m+t$.

This may be a more elegant way of expressing the valuation problem; however, in terms of a practical procedure, it does not move us any closer to a solution. For a valuation computation to proceed it is obvious there must be some kind of numerical substitution for $c^*_{m+1}, c^*_{m+2} \ldots c^*_{m+t}$.

To accomplish this we need to return to fundamentals. If, for example, we do not possess the particular realized price of comparable $m+i$ then we need to think in terms of a likely range of prices which include the unknown c^*_{m+i}. In the light of this we may be able to make an estimate of c^*_{m+i}.

Any estimate must represent a 'balance' between two tendencies: to over- or underestimate the particular price c^*_{m+i}. To be specific, and at the same time accommodate the possibility that the estimate is exactly equal to c^*_{m+i}, we face two possible formulations: a balance between not overestimating and overestimating; or, alternatively, a balance between not underestimating and underestimating. When presented in this manner, comparable $m+i$ in the former case is expressed as $\leqslant c^*_{m+i}$ or $> c^*_{m+i}$. In the latter case the same comparable would be expressed as $\geqslant c^*_{m+i}$ or $< c^*_{m+i}$. The selection between these depends to a certain extent on how the valuation problem has been formulated; we have presented it as one of maximization. Hence the most secure form, in terms being likely to produce a bounded solution space, is of the $\leqslant c^*_{m+i}$ type (how the possibility of overestimation, $> c^*_{m+i}$, fits into this will be explained later).

Confining ourselves to the formulation involving not overestimating or overestimating, we are still presented with the problem of their particular representation. This is dependent upon a balance of (subjective) probabilities. A detailed explanation of this involves us re-examining the linear programming problem we have posed. We are seeking the values of $p_1, p_2 \ldots p_n$ that maximize the value of subject property, v while simultaneously satisfying the m equality comparable constraints, for which the realized prices are known. The additional task which $p_1, p_2 \ldots p_n$ now have to perform is that they should simultaneously be such that overestimation of, say, c^*_{m+i}, should not occur according to some minimum probability, such as α_{m+i}.

In summary we replace the constraint

$$\sum_{j=1}^{n} p_j k_{m+i\,j} = c^*_{m+i} \quad (6)$$

with a probability (Prob) statement in the form of a chance constraint:

$$\text{Prob}\left(\sum_{j=1}^{n} p_j k_{m+i\,j} \leqslant c^*_{m+i}\right) \geqslant \alpha_{m+i} \quad (7)$$

The latter implies that the input characteristic prices which maximize the value of v should be such that the $m+i$ unknown realized price should not be overestimated with at least a minimum probability of α_{m+i}. A corollary of this is that there is a complementary maximum probability of $1-\alpha_{m+i}$ that the property characteristic prices will overestimate the realized price of comparable $_{m+i}$; that is:

$$\text{Prob}\left(\sum_{j=1}^{n} p_j k_{m+i\,j} > c^*_{m+i}\right) \leqslant 1-\alpha_{m+i} \quad (8)$$

4. Imperfect price information and mathematical programming

In Section 3 we have seen how a comparable sale, with an unknown realized sale price, can be expressed as a probabilistic statement. At the extreme all our comparable information may be of this kind, with t unknown prices. If this is so, and we are seeking the maximum valuation of the subject property consistent with the unknown comparable sales prices, the valuation can be represented as a problem in chance-constrained programming (see, for example, Vajda, 1972). Such a problem may take the form:

$$\text{maximize: } \sum_{j=1}^{n} p_j k_{vj} \quad (9)$$

$$\text{subject to: } \left.\begin{array}{l} \text{Prob}\left(\sum_{j=1}^{n} p_j k_{1j} \leqslant c^*_1\right) \geqslant \alpha_1 \\ \text{Prob}\left(\sum_{j=1}^{n} p_j k_{2j} \leqslant c^*_2\right) \geqslant \alpha_2 \\ \cdots \\ \cdots \\ \cdots \\ \text{Prob}\left(\sum_{j=1}^{n} p_j k_{tj} \leqslant c^*_t\right) \geqslant \alpha_t \\ p_1, p_2 \ldots p_t \geqslant 0 \end{array}\right\} \quad (10)$$

The interpretation of this is as follows. We seek the input property characteristic prices (p_1,

$p_2 \ldots p_t$) such that the value of property v is at a maximum, while each comparable's estimated value does not exceed its unknown realized price with a specified minimum probability. This, however, is a somewhat pessimistic formulation of the valuation problem. It is likely that there are some comparables when the realized prices are known. Assume there are m comparables for which this is the case, with prices $c_1, c_2 \ldots c_m$; in addition there are t where the realized prices ($c^*_{m+1}, c^*_{m+2} \ldots c^*_{m+t}$) are unknown. Hence the valuation problem is now:

$$\text{maximize: } \sum_{j=1}^{n} p_j k_{vj} \tag{11}$$

$$\left.\begin{aligned} &\text{subject to: } \sum_{j=1}^{n} p_j k_{1j} = c_1 \\ &\sum_{j=1}^{n} p_j k_{2j} = c_2 \\ &\ldots \\ &\ldots \\ &\sum_{j=1}^{n} p_j k_{mj} = c_m \\ &\text{Prob}\left(\sum_{j=1}^{n} p_j k_{m+1\,j} \leqslant c^*_{m+1}\right) \geqslant \alpha_{m+} \\ &\text{Prob}\left(\sum_{j=1}^{n} p_j k_{m+2\,j} \leqslant c^*_{m+2}\right) \geqslant \alpha_{m+} \\ &\ldots \\ &\ldots \\ &\text{Prob}\left(\sum_{j=1}^{n} p_j k_{m+t\,j} \leqslant c^*_{m+t}\right) \geqslant \alpha_{m+t} \\ &p_1, p_2, \ldots p_n \geqslant 0 \end{aligned}\right\} \tag{12}$$

It should be noted that Equations 11 and 12, like Equation 9 and 10, describe a nonlinear problem.

Obviously the interpretation of this formulation is somewhat different from the earlier one, where all the price data were chance-constrained. We now seek the input property characteristics prices such that the value of property v is maximized while remaining consistent with m certain realized prices and t uncertain prices, where the latter are only exceeded with implied maximum levels of probability ($1-\alpha_{m+1}, 1-\alpha_{m+2} \ldots 1-\alpha_{m+t}$). In this formulation the motive continues to be to improve the valuation by including in the appraisal information in the t chance constraints as well as that in the m certain constraints. However, it should still be borne in mind that we are working with much less information than would be considered necessary for a meaningful econometric-based valuation. In other words $m+t$ is likely to be a small number; for example, not more than six. However, because we now incorporate additional (admittedly probabilistic) information, the quality of the valuation is, we hope, enhanced.

5. Mathematical programming with chance constraints

To this point we have merely sought to argue that probabilistic, as well as certain, information can in principle be incorporated into a programming valuation formulation. However we have not addressed the practice.

The essence of the procedure we are about to describe is to substitute in the place of a unknown realized price, such as c^*_{m+i}, a price reflecting the specified probability level, α_{m+i}. For this to be discovered we need to consider the likely values of c^*_{m+i}. Hence it is necessary to specify its statistical distribution.

Assume c^*_{m+i} is distributed such that:

$$\text{Prob } (c^*_{m+i} \leqslant B) = F(B) \tag{13}$$

Here $F(B)$ is the cumulative probability of the values of c^*_{m+i} up to, and including, $c^*_{m+i} = B$. Of course, implicit in $F(B)$ is the probability density function of c^*_{m+i}; for example, c^*_{m+i} may be uniformly or normally distributed. Clearly the crucial issue is the value of B. Consider the $m+i$ chance constraint (Equation 7 above). From our distributional assumption it is obvious that

$$F(B) = \alpha_{m+i} \tag{14}$$

Hence if we write B_α, it is to be understood that the value B is associated with cumulative probability α (for the moment the subscript $m+i$ associated with it is suppressed in the interests of notational simplicity).

It is tempting to substitute for the $m+i$ constraint the following:

$$\sum_{j=1}^{n} p_j k_{m+i\,j} \leqslant B_\alpha \tag{15}$$

The constraint continues to be specified in terms of avoiding overestimation. However the right-hand side is considerably simplified by specifying a quantity, B_α, to embody the unknown realized price, c^*_{m+i}, and the probability to which we are working. Nevertheless this form of the constraint will not perform the task we require of it. If the constraint is adhered to all we can say is that

$$\sum_{j=1}^{n} p_j k_{m+i\,j} \tag{16}$$

does not exceed, but might be smaller than, those possible values of c^*_{m+i} larger than B_α. However the probability of $c^*_{m+i} > B_\alpha$ is $1-\alpha$. On the other hand, consider the constraint

$$\sum_{j=1}^{n} p_j k_{m+i\,j} \leqslant B_{1-\alpha} \tag{17}$$

The right hand side of Equation 17 is simply the quantile, B, associated with the complementary probability $1-\alpha$. If this version of the $m+i$ constraint holds, Expression 16 cannot be larger, but might be smaller than those possible values of c^*_{m+i} larger than $B_{1-\alpha}$. The probability of $c^*_{m+i} > B_{1-\alpha}$ is α, which is the minimum probability with which we specified the original $m+i$ constraint should comply. Thus the greater the probability, α, with which we wish to avoid overestimating, the lower is the substitute numerical value, $B_{1-\alpha}$. We use $B_{1-\alpha}$ since this permits a probability of at least α that the unknown realized price, c^*_{m+i}, was greater than it. Constraints, of a similar form, are constructed for the remaining $t-1$ comparables with unknown realized prices.

Instead of Equations 11 and 12 we can now solve the linear deterministic equivalent:

$$\text{maximize: } \sum_{j=1}^{n} p_j k_{vj} \tag{18}$$

$$\text{subject to: } \left.\begin{aligned} &\sum_{j=1}^{n} p_j k_{1j} = c_1 \\ &\sum_{j=1}^{n} p_j k_{2j} = c_2 \\ &\quad \cdots \\ &\sum_{j=1}^{n} p_j k_{m+1\,j} = c_m \\ &\sum_{j=1}^{n} p_j k_{m+1\,j} \leqslant B_{1-\alpha_{m+1}} \\ &\sum_{j=1}^{n} p_j k_{m+2\,j} \leqslant B_{1-\alpha_{m+2}} \\ &\quad \cdots \\ &\sum_{j=1}^{n} p_j k_{m+t\,j} \leqslant B_{1-\alpha_{m+t}} \\ &p_1, p_2 \ldots p_n \geqslant 0 \end{aligned}\right\} \tag{19}$$

In the simplest case $\alpha_{m+1} = \alpha_{m+2} \ldots = \alpha_{m+t} = \alpha$.

The immediate problem, of course, is how do we discover the values of all $B_{1-\alpha_{m+i}}$ $(i=1, 2 \ldots t)$? To meet this challenge we need to move on from the statement that an unknown realized property price, such as c^*_{m+i}, is a random variable to the assertion that it possesses a specific statistical distribution. Attention here will be confined to the uniform distribution (others, including the normal distribution, will occur to the reader). This is characterized by two key parameters: it is critically dependent upon its minimum (that is, floor) and maximum (ceiling) values.

In the case of comparable property $m+i$, with unknown realized price c^*_{m+i}, the valuer has to specify, the case of a uniform distribution, a floor price, ${}_f c^*_{m+i}$, and a ceiling price ${}_g c^*_{m+i}$. The judgement is made in the expectation that

$$\text{Prob}({}_f c^*_{m+i} \leqslant c^*_{m+i} \leqslant {}_g c^*_{m+i}) = 1 \tag{20}$$

Further, in the range ${}_f c^*_{m+i}$ to ${}_g c^*_{m+i}$, it is assumed the probability does not change as c^*_{m+i} is lowered and approaches ${}_f c^*_{m+i}$, or is increased towards ${}_g c^*_{m+i}$. Of course the corollary is that prices below ${}_f c^*_{m+i}$, and above ${}_g c^*_{m+i}$, are assumed to be impossible. I suspect that this is a fairly accurate picture as to how a 'practical' valuer would conceptualize the issues. In its discrete guise it appeals through the analogy of the throw of a dice. The latter has six different values, with a floor of one and a ceiling of six. Values outside these are impossible, and every value in the range one to six is equally likely. Superficially, at least, a property price may be thought by the valuer to reflect this schema.

We have yet to determine the linear deterministic equivalent value of c^*_{m+i}, where the latter is assumed to be a random variable with a uniform distribution. Assume, for example, the minimum probability we are working to for the $m+i$ constraint is α_{m+i}. The relevant value of $B_{1-\alpha_{m+i}}$ for a uniform distribution is such that:

$$B_{1-\alpha_{m+i}} = {}_f c^*_{m+i} + ({}_g c^*_{m+i} - {}_f c^*_{m+i})(1-\alpha_{m+i}) \tag{21}$$

Thus the $m+i$ constraint, Equation 7, is re-expressed as

$$\sum_{j=1}^{n} p_j k_{m+i\,j} \leqslant B_{1-\alpha_{m+i}} = {}_f c^*_{m+i} + ({}_g c^*_{m+i} - {}_f c^*_{m+i})(1-\alpha_{m+i}) \tag{22}$$

All the chance constraints are converted in this manner, with the appropriate substitutions being made for each comparable as to the values of its ceiling price, ${}_g c^*$, floor price, ${}_f c^*$, and the minimum probability, α, being worked to.

A consequence of treating the unknown realized prices as random variables is that we need to consider the explicity the relationship between them. In the linear programming formulation it is assumed that the unknown prices are statistically independent of each other. The implication of this is that, using the example of two comparables with unknown realized prices, the probability of any pair of given price ranges occurring is equal to the product of their marginal probabilities. Further, where the unknown prices each have a uniform probability distribution, it is assumed any given pair of price ranges, within the respective overall floor and ceiling prices of the two comparables, are equally probable. There are additional implications of the assumption of a uniform probability distribution. It is bounded at both its upper and lower ends. Those bounds, the ceiling and floor prices, reflect the valuer's assessment of current market conditions. Prices outside the bounds can only be achieved, in the valuer's opinion, following a change in the market. In other words, all properties being used to carry out the valuation would cease to be comparables. Thus the valuer is forced to use overall price ranges which are consistent with each other in the context of the current market.

There is a further implication of the assumed statistical independence of the unknown realized prices. As the number of comparables with such prices increases, the probability of at least one being breached (that is, an unknown price being overestimated) unambiguously increases. The probability of this is equal to the following, where Π is the product symbol:

$$1 - \prod_{i=m+1}^{i=m+t} (\alpha_i) \tag{23}$$

Hence, if we have two comparables with unkown realized prices, and each separately is set to avoid overestimation at a probability of 97.5%, the probability of there being at least one overestimation is, using expression 23, nearly 5%. There are, of course, two constraints on this 'error'. These are the levels of probability we choose to work with, and the number of comparables with unknown realized prices that can be utilized in the valuation. The latter is likely to be small bearing in mind the context we are working in; further, we can work to whatever level of probability we choose. Of course this may still be deemed to be unsatisfactory. The solution in that case is not to specify separately the probability for each of the t comparables' chance constraints being observed. Instead the comparables should be presented such that there is an overall declared level of probability that they are not breached. This, however, takes us into the field of a nonlinear programming and outside the scope of this particular paper.

6. An example

It may assist readers to consolidate their understanding of the above if we work through an example. Assume we wish to value propert v. We possess data on three comparable properties, which have been sold recently (x, y, z). For all four we have data on five property characteristics. The reader may wish to think of these in specific terms. In the case of a residential valuation the five characteristics may be imagined as:

1. gross external area, in square metres;
2. number of bedrooms;
3. presence (=1) or absence (=0) of a garage;
4. presence (=1) or absence (=0) of central heating;
5. area of garden, in square metres.

In addition we possess the realized selling price of property z (£53 000). We have no specific prices for the other two comparables. However the valuer is 'convinced' that floor and ceiling prices can be identified within which ranges lie their particular selling prices (c_x^*, c_y^*). We assume further that both c_x^* and c_y^* are random variables with uniform distributions. Their respective floor and ceiling prices, together with the other data, on the four properties are presented in Table 1; the floor and ceiling price columns are straddled in the case of property z since the selling price is known with certainty.

Table 1.

Comparable properties	Property characteristics[a] (i)	(ii)	(iii)	(iv)	(v)	Floor price[a]	Ceiling price[a]
x	105	3	0	1	140	45 000	49 000
y	101	2	0	0	115	40 000	46 000
z	89	3	1	0	170	53 000	
Property to be valued							
v	109	3	0	1	160		

[a]The units used in Table 1 are described in the text above.

The problem is to find the maximum value of property v consistent with the price of property z equalling £53 000, and there being, say, a 97.5% probability that such a valuation of v is consistent with, respectively, neither c_x^* nor c_y^* being exceeded. The mathematical programme to be solved is as follows:

$$\text{maximize: } 109p_1+3p_2+p_4+160p_5 \tag{24}$$

$$\left.\begin{aligned}\text{subject to: } &\text{Prob}\,(105p_1+3p_2+p_4+140p_5\leqslant c_x^*)\geqslant 0.975\\ &\text{Prob}\,(101p_1+2p_2+115p_5\leqslant c_y^*)\geqslant 0.975\\ &89p_1+3p_2+p_3+170p_5=53\,000\\ &p_1,\,p_2,\,p_3,\,p_{43},\,p_5\geqslant 0\end{aligned}\right\} \tag{25}$$

This converts into the linear programming equivalent:

$$\text{maximize: } 109p_1+3p_2+p_4+160p_5 \tag{26}$$

$$\left.\begin{aligned}\text{subject to: } &105p_1+3p_2+p_4+140p_5\leqslant 45\,000(-45\,000)\,0.025\\ &101p_1+2p_2+115p_5\leqslant 40\,000+(46\,000-40\,000)\,0.025\\ &89p_1+3p_2+p_3+170p_5=53\,000\\ &p_1,\,p_2,\,p_3,\,p_4,\,p_5\geqslant 0\end{aligned}\right\} \tag{27}$$

On the basis of this formulation we find, having used the simplex algorithm, that the value of property v is approximately £51 300.

7. Conclusions

Superficially it may appear only too easy to criticize a chance-constrained programming approach to valuation. Surely all it amounts to is, in addition, we have to estimate the selling price of one, or worse more, properties, that have been sold in order to value the particular one that has not. How can this contribute to improved decision-making? Is it not better to remain with the primary task of valuing the one property that has to be valued?

Any debate of the key issues must commence with the primary constraining fact of valuation by comparable sales: that there is very little information available to carry out the task (hence an econometric approach is precluded). However, it has been implicitly assumed that what is available can be completely relied upon. The possibility arises that we have overstated the number of comparables, for which this is the case. If so, the question then becomes: is there any other information that could be utilized? This paper has suggested that we incorporate into the analysis those sales whose realized prices are not known. Hence statistical information is incorporated explicitly into the valuation; however this is processed from the point of view of achieving algebraic and arithmetical consistency rather than in the context of inferential econometric analysis.

There are at least two arguments in support of the chance-constrained approach. The first is factual and the second methodological. Even if we do not know the price a comparable property realized recently we do know its characteristics are in demand and have been implicitly priced and exchanged in the market. Where facts are scarce this information is a leading candidate to be incorporated into the valuation. However, when incorporating such information we have to avoid falling into the seme error as traditional valuation by comparable sales. This has been criticized for the *ad hoc* nature of its methodology. However, a chance-constrained programming approach to valuation attempts to avoid this. The formulation of the problem must meet the strictures of a linear programme. The statistical distribution of the unknown prices must be declared, and the level, or levels, of probability being worked to also need to be specified. It is not denied that subjective information is incorporated; tht is inevitable in the face of a scarcity of market information. However, the incorporation is carried out explicitly, thereby facilitating principled and objective debate.

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