MATH 340 Practice for Quiz # 2 on Friday Jan. 19.

1. Solve the following LP using our Two Phase Method.

$$\begin{array}{rrrrl} \text{Maximize} & 2x_1 & +5x_2 & +4x_3 & \\ & x_1 & -x_2 & +x_3 & \le 1 \\ & -x_1 & -x_2 & & \le -3 \\ & & -x_2 & -x_3 & \le -2 \end{array} \qquad x_1, x_2, x_3 \ge 0$$

Solution:

Phase One:

$$\begin{array}{rcrrrrr} x_4 & = & 1 & -x_1 & +x_2 & -x_3 & +x_0 \\ x_5 & = & -3 & +x_1 & +x_2 & & +x_0 \\ x_6 & = & -2 & & +x_2 & +x_3 & +x_0 \\ w & = & & & & & -x_0 \end{array}$$

x_0 enters and x_5 leaves (Fake pivot to feasibility)

$$\begin{array}{rcrrrrr} x_4 & = & 4 & -2x_1 & & -x_3 & +x_5 \\ x_0 & = & 3 & -x_1 & -x_2 & & +x_5 \\ x_6 & = & 1 & -x_1 & & +x_3 & +x_5 \\ w & = & -3 & +x_1 & +x_2 & & -x_5 \end{array}$$

x_1 enters and x_6 leaves

$$\begin{array}{rcrrrrr} x_4 & = & 2 & +2x_6 & & -3x_3 & -x_5 \\ x_0 & = & 2 & +x_6 & -x_2 & -x_3 & \\ x_1 & = & 1 & -x_6 & & +x_3 & +x_5 \\ w & = & -2 & -x_6 & +x_2 & +x_3 & \end{array}$$

x_2 enters and x_0 leaves

$$\begin{array}{rcrrrrr} x_4 & = & 2 & +2x_6 & & -3x_3 & -x_5 \\ x_2 & = & 2 & +x_6 & -x_0 & -x_3 & \\ x_1 & = & 1 & -x_6 & & +x_3 & +x_5 \\ w & = & 0 & & -x_0 & & \end{array}$$

End of Phase One. Delete x_0, w and introduce z. Note $z = 2x_1 + 5x_2 + 4x_3 = 2(1 - x_6 + x_3 + x_5) + 5(2 + x_6 - x_3) + 4x_3$.

$$\begin{array}{rcrrrr} x_4 & = & 2 & +2x_6 & -3x_3 & -x_5 \\ x_2 & = & 2 & +x_6 & -x_3 & \\ x_1 & = & 1 & -x_6 & +x_3 & +x_5 \\ z & = & 12 & +3x_6 & +x_3 & +2x_5 \end{array}$$

x_6 enters and x_1 leaves

$$\begin{array}{rcrrrr} x_4 & = & 4 & -2x_1 & -x_3 & +x_5 \\ x_2 & = & 3 & -x_1 & & +x_5 \\ x_6 & = & 1 & -x_1 & +x_3 & +x_5 \\ z & = & 15 & -3x_1 & +4x_3 & +5x_5 \end{array}$$

x_5 enters but no leaving variable. The LP is unbounded with feasible solutions $(0, 3+t, 0, 4+t, t, 1+t)$ for $t \geq 0$ with $z = 15 + 5t$.

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1. Solve the following LP using our Two Phase Method. (only one such question will be asked in the quiz; I've given you two samples)

$$\begin{array}{lrrrl} \text{Maximize} & -2x_1 & & -x_3 & \\ & -x_1 & -x_2 & -x_3 & \leq 3 \\ & -x_1 & & -x_3 & \leq -2 \\ & -2x_1 & -x_2 & -x_3 & \leq -3 \end{array} \qquad x_1, x_2, x_3 \geq 0$$

Solution:

Phase One:

$$\begin{array}{rcrrrrr} x_4 & = & 3 & +x_1 & +x_2 & +x_3 & +x_0 \\ x_5 & = & -2 & +x_1 & & +x_3 & +x_0 \\ x_6 & = & -3 & +2x_1 & +x_2 & +x_3 & +x_0 \\ w & = & & & & & -x_0 \end{array}$$

x_0 enters and x_6 leaves (Fake pivot to feasibility)

$$\begin{array}{rcrrrrr} x_4 & = & 6 & -x_1 & & & +x_6 \\ x_5 & = & 1 & -x_1 & -x_2 & & +x_6 \\ x_0 & = & 3 & -2x_1 & -x_2 & -x_3 & +x_6 \\ w & = & -3 & +2x_1 & +x_2 & +x_3 & -x_6 \end{array}$$

x_1 enters and x_5 leaves

$$\begin{array}{rcrrrrr} x_4 & = & 5 & +x_5 & +x_2 & & \\ x_1 & = & 1 & -x_5 & -x_2 & & +x_6 \\ x_0 & = & 1 & +2x_5 & +x_2 & -x_3 & -x_6 \\ w & = & -1 & -2x_5 & -x_2 & +x_3 & +x_6 \end{array}$$

x_3 enters and x_0 leaves

$$\begin{array}{rcrrrrr} x_4 & = & 5 & +x_5 & +x_2 & & \\ x_1 & = & 1 & -x_5 & -x_2 & & +x_6 \\ x_3 & = & 1 & +2x_5 & +x_2 & -x_0 & -x_6 \\ w & = & 0 & & & -x_0 & \end{array}$$

End of Phase One. Delete x_0, w and introduce z. Note $z = -2x_1 - x_3 = -2(1 - x_5 - x_2 + x_6) - (1 + 2x_5 + x_2 - x_6) = -3 + x_2 - x_6$.

$$\begin{array}{rcrrrr} x_4 & = & 5 & +x_5 & +x_2 & \\ x_1 & = & 1 & -x_5 & -x_2 & +x_6 \\ x_3 & = & 1 & +2x_5 & +x_2 & -x_6 \\ z & = & -3 & & +x_2 & -x_6 \end{array}$$

x_2 enters and x_1 leaves

$$\begin{array}{rcrrrr} x_4 & = & 6 & & -x_1 & +x_6 \\ x_2 & = & 1 & -x_5 & -x_1 & +x_6 \\ x_3 & = & 2 & +x_5 & -x_1 & \\ z & = & -2 & -x_5 & -x_1 & \end{array}$$

Optimal solution: $(0,1,2,6,0,0)$ with $z=-2$. Note that $(0,1+t,2,6+t,0,t)$ yields additional optimal solutions for $t>0$.

The intent of the quiz is to familiarize yourself with the entire simplex algorithm and you will be expected to answer a similar question on both the midterm and the final. The second example above pivots to an optimal solution as will the Quiz 2 question you will be given (you will be given the optimal value of z as a check for your work). You must also know (but not for Quiz 2) what happens if there is no feasible solution (thus Phase One terminates with $x_0>0$) or the LP is unbounded (and in that case you should report a parametric set of feasible solutions whose z values $\longrightarrow\infty$ as the parameter $\longrightarrow\infty$ exactly as done in the first example above).

There is a fair amount of number crunching (mostly with small integers) but you are exposed to the standard set of pivots. You will be able to handle the sensitivity analysis questions later more easily later. If, for some reason, such number crunching questions for quizzes are too much for you then if you contact me early on (before the quiz 2) we could discuss other grading schemes avoiding the quizzes. You would nonetheless be expected to do the number crunching questions on the midterm and the final exam.

You can check your work by testing for the feasibility of your final answer by substituting in the original problem and also checking the value of z by substituting in its original expression. If you find an error please make a note of it for me since you may get marks for noticing that you have made an error.