1. Put the following LP in Standard Inequality Form.

$$
\begin{array}{lllll}
\text{Minimize} & 2x_1 & +4x_2 & & +6 \\
 & x_1 & & +x_3 & \geq 4 & x_2, x_3 \geq 0; \quad x_1 \text{ free} \\
 & & x_2 & +x_3 & = 1
\end{array}
$$

2. Using the following dictionary and Anstee's Rule, perform two pivots to get to an optimal solution. Report the optimal solution.

$$
\begin{array}{rllll}
x_1 & = 2 & +x_4 \\
x_2 & = 1 & -x_4 & +x_5 \\
x_3 & = 3 & -x_4 \\
z & = -3 & +2x_4 & -x_5
\end{array}
$$

---

1. Solution:

$$
\begin{array}{lllll}
\text{Minimize} & 2x_1 & +4x_2 & & +6 \\
 & x_1 & & +x_3 & \geq 4 & x_2, x_3 \geq 0; \quad x_1 \text{ free} \\
 & & x_2 & +x_3 & = 1
\end{array}
$$

minimize $2x_1 + 4x_2 + 6$ yields same optimal solution (of variables) as as maximize $-2x_1 - 4x_2$

$x_1 + x_3 \geq 4 \quad \Longleftrightarrow \quad -x_1 - x_3 \leq -4$

$x_2 + x_3 = 1 \quad \Longleftrightarrow \quad x_2 + x_3 \leq 1 \quad \textbf{and} \quad -x_2 - x_3 \leq -1$

x_1 free (i.e. unrestricted variable, not constrained as to sign) so let $x_1 = x_1' - x_1''$ with $x_1', x_1'' \geq 0$. The new LP becomes

$$
\begin{array}{lllllll}
\text{Maximize} & -2x_1' & +2x_1'' & -4x_2 \\
 & -x_1' & +x_1'' & & -x_3 & \leq -4 \\
 & & & x_2 & +x_3 & \leq 1 & x_1', x_1'', x_2, x_3 \geq 0 \\
 & & & -x_2 & -x_3 & \leq -1
\end{array}
$$

2. Solution

$$
\begin{array}{rllll}
x_1 & = 2 & +x_4 \\
x_2 & = 1 & -x_4 & +x_5 \\
x_3 & = 3 & -x_4 \\
z & = -3 & +2x_4 & -x_5
\end{array}
$$

We choose x_4 to increase since coefficient in z equation is positive. Our rough work, which you don't need to display,is:

$x_1 = 2 + x_4 \to x_4 \geq -2$

$x_2 = 1 - x_4 \to x_4 \leq 1$

$x_3 = 3 - x_4 \to x_4 \leq 3$

and so we increase x_4 to 1 while driving x_2 to 0 and so x_4 enters basis while x_2 leaves.

$$
\begin{array}{rllll}
x_1 & = 3 & -x_2 & +x_5 \\
x_4 & = 1 & -x_2 & +x_5 \\
x_3 & = 2 & +x_2 & -x_5 \\
z & = -1 & -2x_2 & +x_5
\end{array}
$$

We choose x_5 to increase since coefficient in z equation is positive. Our rough work, which you don't need to display,is:

$x_1 = 3 + x_5 \rightarrow x_5 \geq -3$

$x_4 = 1 + x_5 \rightarrow x_5 \geq -1$

$x_3 = 2 - x_5 \rightarrow x_5 \leq 2$

and so we increase x_5 to 2 while driving x_3 to 0 and so x_5 enters basis while x_3 leaves. Do report which variable you are choosing to enter the basis and which variable you are choosing to leave the basis.

$$
\begin{array}{rccl}
x_1 & = 5 & & -x_3 \\
x_4 & = 3 & & -x_3 \\
x_5 & = 2 & +x_2 & -x_3 \\
z & = 1 & -x_2 & -x_3
\end{array}
$$

The coefficients in the final row are all ≤ 0 and so we have an optimal solution: $x_1 = 5, x_4 = 3, x_5 = 2, x_2 = x_3 = 0$ (with $z = 1$). Remember the optimal solution refers to the values for the decision variables.

The intent of question 1 is to familiarize you with some standard transformations and also familiarize you with the fact that an LP can be put in standard form. The main duality theorems will be stated for LP's in this standard form. The simplex method deals with an LP in equality form, not inequality form, obtained by adding slack variables. All the transformations are now typically handled by software packages such as LINDO (note that LINDO does imagine all variables to be positive but you are given the option of unrestricted/free variables by use of the FREE command) and so you would not need to do these transformations for a practical problem.

The intent of question 2 is to familiarize you with the standard pivot process. If you make wrong choices for the entering variable then you might have the z value decrease (or encounter challenging fractions). If you make wrong choices for a leaving variable then you may have a basic variable take on a negative value and the basic solution may no longer be feasible. Notice these errors! Note that we always report the values of the variables at optimality with the current value of the objective function being typically a lesser concern.