Revised Simplex Formulas:

$$\mathbf{x}_B = B^{-1}\mathbf{b} - B^{-1}A_N\mathbf{x}_N$$
$$z = \mathbf{c}_B^T B^{-1}\mathbf{b} + (\mathbf{c}_N^T - \mathbf{c}_B^T B^{-1}A_N)\mathbf{x}_N$$

Original problem:

$$\begin{array}{lccccl}
\text{Maximize} & & 4x_2 & +2x_3 & & \\
& x_1 & +2x_2 & +x_3 & & \leq 3 \\
& x_1 & -x_2 & +2x_3 & +x_4 & \leq 3 \\
& & +x_2 & & x_4 & \leq 1
\end{array} \qquad x_1, x_2, x_3, x_4 \geq 0$$

$$\mathbf{c}^T = \begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \end{matrix} \\ \begin{pmatrix} 0 & 4 & 2 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

$$A = \begin{array}{c} \\ x_5 \\ x_6 \\ x_7 \end{array} \begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \end{matrix} \\ \begin{pmatrix} 1 & 2 & 1 & 0 & 1 & 0 & 0 \\ 1 & -1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \end{array} \quad \begin{array}{c} \\ x_5 \\ x_6 \\ x_7 \end{array} \begin{array}{c} b \\ \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix} \end{array}$$

Let us claim that $\{x_2, x_3, x_4\}$ is an optimal basis. We first compute the associated B^{-1} for convenience.

$$\text{basis: } \{x_2, x_3, x_4\},\ B^{-1} = \begin{array}{c} \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{array}{c} \begin{matrix} x_5 & x_6 & x_7 \end{matrix} \\ \begin{pmatrix} 1/3 & -1/6 & 1/6 \\ 1/3 & 1/3 & -1/3 \\ -1/3 & 1/6 & 5/6 \end{pmatrix} \end{array}$$

Final dictionary (for basis $\{x_2, x_3, x_4\}$) computed by Revised Simplex Formulas

$$\begin{array}{ccccccc}
x_2 & = & 2/3 & -1/6x_1 & -1/3x_5 & +1/6x_6 & -1/6x_7 \\
x_3 & = & 5/3 & -2/3x_1 & -1/3x_5 & -1/3x_6 & +1/3x_7 \\
x_4 & = & 1/3 & +1/6x_1 & +1/3x_5 & -1/6x_6 & -5/6x_7 \\
z & = & 6 & -2x_1 & -2x_5 & &
\end{array}$$

We read off the values 2, 0, 0 as the negatives of the coefficients of the slack variables, and deduce these are an optimal dual solution.

sample computation by revised simplex formulas

$$\begin{array}{cc} & \begin{array}{cccc} x_1 & x_5 & x_6 & x_7 \end{array} \\ \mathbf{c}_N^T = & \begin{pmatrix} 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

$$\mathbf{c}_B^T B^{-1} A_N = (\,4 \quad 2 \quad 0\,) \begin{pmatrix} 1/3 & -1/6 & 1/6 \\ 1/3 & 1/3 & -1/3 \\ -1/3 & 1/6 & 5/6 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= (\,2 \quad 0 \quad 0\,) \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = (\,2 \quad 2 \quad 0 \quad 0\,)$$

Thus

$$\begin{array}{cc} & \begin{array}{cccc} x_1 & x_5 & x_6 & x_7 \end{array} \\ \mathbf{c}_N^T - \mathbf{c}_B^T B^{-1} A_N = & \begin{pmatrix} -2 & -2 & 0 & 0 \end{pmatrix} \end{array}$$

Why are the negatives of the coefficients of the slack variables in the z row of a dictionary equal to $\mathbf{c}_B^T B^{-1}$? Here is the reason:

$$z = \mathbf{c}_B^T B^{-1}\mathbf{b} + (\mathbf{c}_N^T - \mathbf{c}_B^T B^{-1} A_N)\mathbf{x}_N$$

$$= \mathbf{c}_B^T B^{-1}\mathbf{b} + ((\mathbf{c}^T\ \mathbf{0}^T) - \mathbf{c}_B^T B^{-1}[A\, I]) \begin{pmatrix} \mathbf{x} \\ \mathbf{x}_S \end{pmatrix}$$

(here we add the trivial entries $0 = (c_B^T - c_B^T B^{-1} B)\mathbf{x}_B$ and then shuffle the variables into the original variables and the slack variables)

$$= \mathbf{c}_B^T B^{-1}\mathbf{b} + (\mathbf{c}^T - \mathbf{c}_B^T B^{-1} A)\mathbf{x} + (-c_B^T B^{-1})\mathbf{x}_S$$

You may check that our solution $(2, 0, 0)$ is optimal in the dual LP:

$$\begin{array}{lrrrll} \text{Minimize} & 3y_1 & +3y_2 & +y_3 & & \\ & y_1 & +y_2 & & \geq 0 & \\ & 2y_1 & -y_2 & +y_3 & \geq 4 & \qquad y_1, y_2, y_3 \geq 0 \\ & y_1 & +2y_2 & & \geq 2 & \\ & & y_2 & +y_3 & \geq 0 & \end{array}$$