Math 104/184 Curve Sketching Richard Anstee, November 2006.

The book describes the process on page 281. This is just my version of the process.
1. Identify the domain and range of the function. For example determine points where the function is not defined (such as because of division by 0) or not continuous. It would be nice to compute where the curve crosses the y-axis (by computing $f(0)$) and crosses the x-axis (by finding x for which $f(x) = 0$) and then determing a sign chart for f, namely the intervals where $f(x) > 0$ and intervals where $f(x) < 0$.
2. Look for asymptotes. One looks for horizontal asymptotes by computing $\lim_{x \to +\infty} f(x)$, $\lim_{x \to -\infty} f(x)$. If the limit for $x \to \infty$ is a constant L then $y = L$ is a horizontal asymptote as $x \to \infty$. If the limit for $x \to -\infty$ is a constant L then $y = L$ is a horizontal asymptote as $x \to -\infty$. There could be curves with two different horizontal asymptotes.

To look for vertical asymptotes, consider values $x = a$ where the function is undefined or not continuous. Then if $\lim_{x \to a^-} f(x) = \pm\infty$ or $\lim_{x \to a^+} f(x) = \pm\infty$, then we have a vertical asymptote at $x = a$.
3. Compute $f'(x)$. Determine a sign chart for $f'(x)$. Identify critical points and perhaps use the first derivative test to identify any local maximums or local minimums. Identify intervals where the function is increasing ($f'(x) > 0$) and intervals where the function is decreasing ($f'(x) < 0$).
4. Compute $f''(x)$. Determine a sign chart for $f''(x)$. Identify inflection points ($f''(x) = 0$). Identify intervals where the function is concave up ($f''(x) > 0$) and intervals where the function is concave down ($f''(x) < 0$).
5. Notes: In determining facts about $f(x), f'(x), f''(x)$, it will be helpful to 'simplify' perhaps by pulling out common factors etc. The payoff to simplifying first and second derivatives will be ease in determining the sign charts and to some extent ease in computing the second derivative from the first derivative.

Students are often confused by concavity. A function can be concave up on an interval (namely derivative is increasing on that interval), without ever actually going up (i.e. derivative might remain negative even though it is increasing since the rate of increase of the derivative ($f''(x)$) gets smaller and smaller.

In displaying your answer, you will obviously be displaying your first and second derivatives (including any places where they are not defined) as well as sign charts for $f(x), f'(x), f''(x)$. You would label x and y axes as well. Graphing some important points (where curve crosses axes, critical points, inflection points) and sketching the curve. The accuracy of your plot is less crucial than communicating the 'shape' of the curve. Thus if you identify an inflection point at $(2, 3.2)$ I wouldn't be so critical about the placement of the point but would like to see some evidence of the change in concavity at that point in your sketch.