

Mock Exam: Long Answer Questions

Time: 50 minutes

1. 4 marks (2014) Let $g(x) = \frac{x}{2+3x}$. Compute $g'(x)$ using the definition of the derivative. **No marks will be given for use of derivative rules**, but you may use them to check your answer.
2. (2014)

 - (a) 4 marks Let $f(x) = e^x - e^{-x}$. Find the global maximum and minimum of $f(x)$ on the interval $[-1, 1]$.
 - (b) 5 marks Now let $g(x) = e^x + e^{-x}$. Your friend uses a degree-2 Taylor polynomial about $x = 0$ to approximate $g(1)$. Show that the error in their approximation is no more than $4/9$.
3. 8 marks (2013) A right circular cylinder is inscribed in a sphere of radius 3. Find the largest possible volume of such a cylinder.
4. 15 marks (2015) Let $f(x) = (x+1)(x+2)e^{-x}$.

 - (a) 1 mark Find the domain of the function.
 - (b) 2 marks Find the x -intercepts and the y -intercepts of the function
 - (c) 2 marks Find the horizontal asymptotes of the function (if they exist)
 - (d) 1 mark Find the vertical asymptotes of the function (if they exist)
 - (e) 1 mark Find $f'(x)$. You must simplify your answer.
 - (f) 2 marks Find all critical points and singular points of $f(x)$.
 - (g) 3 marks Find the intervals on which $f(x)$ is increasing. Find the intervals on which $f(x)$ is decreasing.
 - (h) 2 marks The second derivative is $f''(x) = (x-2)(x+1)e^{-x}$. Find the x -coordinates of any points of inflection. You must explain your answer.
5. 7 marks (2013) A spectator is 4 km from a launch site as a rocket is being launched. When the rocket is 3 km high, it is moving directly upward at a rate of 0.7 km per second. How fast is the distance between the spectator and the rocket increasing at that instant?