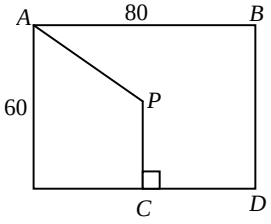
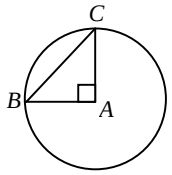
UBC Grade 8–10 Workshop Problems, 2009–2010

1. James leaves a point *A* at 7:00 a.m. and travels at 30 km/h. An hour later, Kevin leaves the same point *A* and travels along the same road, but at 45 km/h. At what time will Kevin meet James?

2. In a rectangular town that has length 80 km and width 60 km, there is a park, *P*, in the centre of the town. Two friends, David and Caitlyn, plan to meet at this park. David leaves his house at point *B* and follows the path from *B* to *A* to *P*, while Caitlyn leaves her house at point *D* and follows the path from *D* to *C* to *P*. If they both leave at the same time and David travels at 65 km/h, how fast does Caitlyn need to travel to get to the park at the same time as David?

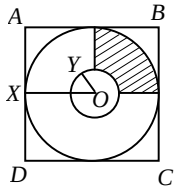


3. In the picture, *A* is the centre of the circle, triangle *ABC* is a right triangle, and *B* and *C* are on the circle. If the area of the circle is 36π, what is the length of *BC*?



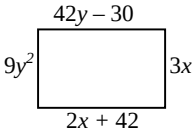
4. John writes 11 assignments for a math course and his average mark is 68%. His teacher drops the lowest assignment mark, which makes the new average 71%. What was the mark on the assignment that was dropped?

5. *ABCD* is a square. Within this square is a circle which intersects the square at the midpoints of its four sides. *XO* is the radius of this circle and is 1 cm. Another circle that is a quarter of the area of the larger circle is placed within the larger circle. Its radius is *OY*. Find the area of the shaded region.



6. A piggy bank holds nickels and dimes. The total value of the coins in the piggy bank is \$4.40, and there are 61 coins in total. How many coins of each type are there?

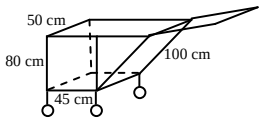
7. The sides of the rectangle in the picture are as given. Not all the lengths of the sides are even numbers. Find the dimensions of the rectangle.



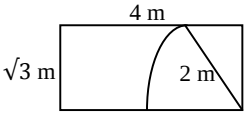
8. 5*x*4*y* is a four-digit number that is exactly divisible by 15. Determine how many such numbers there are.

9. Natasha and her friends drive down to Seattle from Vancouver. Before they reach Seattle they stop at the Premium Outlets along the way. Now, Natasha is a bit of a speeder and drives 110 km/h instead of the 100 km/h speed limit from Vancouver to the Premium Outlets, and 125 km/h from the Premium Outlets to Seattle. Her speeding saves 9 min and 15 min respectively. How far is it from Vancouver to Seattle?

10. Janet, the school janitor, needs to move a 76.8 m^3 pile of gravel from the soccer field to the playground. The dimensions of her wheelbarrow, which is uniformly 50 cm wide, are given in the picture. If Janet only wheels the wheelbarrow when it is exactly full, how many trips will be needed to move the gravel?



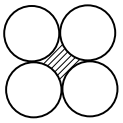
11. Sally the poodle is placed in a rectangular backyard of size 4 m by $\sqrt{3}$ m. Sadly, her owner forgot to untie her leash that is attached to a pole at one corner of the yard. If the leash is 2 m long, what is the area of the region of the yard in which Sally can play?



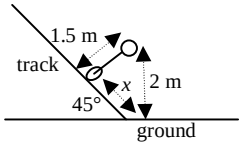
12. Zeus is thinking of a three-digit number. He tells us that the number is odd number, that its digits add up to 15, and that the difference between the first digit and the second digit is equal to the difference between the second digit and the third digit. How many possible such numbers are there?

13. Flash Gordon Junior wants to run as fast as his dad. First he runs across an airport walkway in 4 seconds. He then wants to go faster, so he runs at the same pace and distance but this time on a moving sidewalk, and it take him 3 seconds. How long would it take Junior to run back on the same moving sidewalk, moving against the direction of the moving sidewalk?

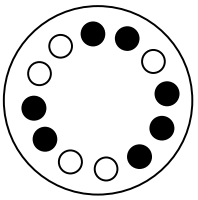
14. Four garden plots which are perfect circles are arranged as shown. Each one is the same size, of radius 1. What is the area of the shaded region that lies in the middle between the garden plots?



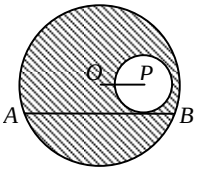
15. Sitting on his bike, Alan is 1.5 m tall. At the bike track, he rides his bike around in a circle. The track is at an angle of 45 degrees to the ground. The warranty on his helmet is void if he crashes with his head farther than 2 m from the ground. If Alan always rides exactly perpendicular to the track, how far up it can he go ("*x*" in the diagram) without voiding the warranty?



16. Jenny has a partially eaten box of chocolates sitting on a Lazy Susan in front of her. The box is in the shape of a circle, with 12 evenly spaced compartments for chocolates. Five of the chocolates have been eaten, but seven remain, configured as shown in the picture (filled circles represent chocolates). Jenny closes her eyes and spins the Lazy Susan, but no chocolate ends up in front of her. Should Jenny spin the Lazy Susan again, hoping that a chocolate now lands in front of her, or simply try the compartment that is to the right of the one that is in front of her?



17. In the diagram, *O* is the centre of the large circle and *P* is the centre of the small circle. The line *AB*, which just touches the smaller circle, is parallel to the line *OP*. If the area of the shaded region, which lies outside the small circle and inside the large one, is 9π, what is the length of *AB*?



18. One summer afternoon at the beach, Abe is bored and starts drawing circles in the sand. He first draws a circle of diameter 1 m. Then he draws a circle of diameter 1/2 m inside the larger circle, just touching the rightmost point of the larger circle. Inside that second circle he draws a circle of diameter 1/4 m just touching the leftmost point of the second circle, and so on. Abe has the feeling that the centres of these circles are getting closer and closer to a single point. How far away is that point from the leftmost point of the original circle? You may use the fact that 1/2 + 1/4 + 1/8 + 1/16 + 1/32 +… = 1.

SOLUTIONS

Note: These concise solutions are meant for workshop leaders and teachers. Presentations to Grade 8–10 students should include additional detail and motivation. The solutions outlined here are considered appropriate for school students at this level; alternate solutions are often possible.

1. Answer: 10:00 a.m. James is 30 km ahead by the time Kevin starts at 8:00 a.m., and Kevin makes up 15 km in 1 h.
2. Answer: 35 km/h. Since AP has length 50 (note the 3-4-5 right triangle), David travels 80 + 50 = 130 km and so takes 2 h. Caitlyn travels 40 + 30 = 70 km and in order to take 2 hours as well must travel at 35 km/h.
3. Answer: $6\sqrt{2}$. The radius is 6 and we have an isosceles right triangle whose hypotenuse is $\sqrt{2}$ times the sides.
4. Answer: 38. The total number of marks for the 11 assignments is 11 x 68 = 748, and for the 10 highest assignments it is 10 x 71 = 710.
5. Answer: $3\pi/16$. The smaller circle, having ¼ the area of the big one, has radius ½ cm. The shaded area is ¼ the difference in areas of these two circles, namely $\pi/4(1^2-(1/2)^2)$.
6. Answer: 34 nickels, 27 dimes. If there were all nickels, then the piggy bank would hold 61 x \$0.05 = \$3.05. The extra \$1.35 means that 135/5 = 27 of the coins are actually dimes.
7. Answer: height 81, width 96. Equating the vertical sides gives us $x = 3y2$, which when substituted into the equated horizontal sides gives the quadratic equation $y^2 - 7y + 12 = 0$, with roots 3 and 4. The solution $y = 4$ results in all the dimensions being even, but $y = 3$ does not.
8. Answer: 7. Divisibility by 15 is equivalent to divisibility by 3 and 5. Divisibility by 5 means y is 0 or 5. Divisibility by 3 occurs when the sum of the digits is a multiple of 3. If $y = 0$ this means $9 + x$ is a multiple of 3, so $x = 0, 3, 6,$ or 9 (4 solutions) and if $y = 5$ then we need $14 + x$ is a multiple of 3 so $x = 1, 4,$ or 7 (3 solutions).
9. Answer: 290 km. If a is the distance from Vancouver to Premium Outlets in km, then the time in h saved by speeding on this leg of the journey is $a/100 - a/110 = a/1100$. Setting this equal to 9/60 = 3/20 yields $a = 165$ km. The distance b from Premium Outlets to Seattle is given by $b/100 - b/125 = 15/60 = ¼$ which gives $b = 125$ km.
10. Answer: 256. The parallel sides of the wheelbarrow have area 45 x 80 + (80 x 60)/2 (note the 3-4-5 right triangle) = 3600 + 2400 = 6000 cm^2. Multiplying by the width 50 gives a wheelbarrow volume of 300000 cm^3 or 0.3 m^3. Now note that 76.8/0.3 = 768/3 = 256. No calculator!
11. Answer: $\sqrt{3}/2 + 2\pi/3$. The triangular portion of Sally's domain is a 30-60-90 right triangle with area $\sqrt{3}/2$. The circular sector is subtended by an angle of 60° and so has area $(\pi 2^{2)}/6 = 2\pi/3$.
12. Answer: 5.The last fact tells us that the sum of the three digits is 3 times the middle digit, hence, by the second fact, the second digit is 5. The common difference of digits must be even in order for the last digit, and hence the number, to also be odd. Letting that common difference by 0, 2, 4, -2, -4 gives 5 such numbers.
13. Answer: 6 seconds. Let d be the length of the walkway, v the unassisted speed of Junior, and w the speed of the walkway. We have $4v = d$ and $3(v + w) = d$ and hence, equating, $w = v/3$. The time t for the retarded run is given by $t(v - w) = d$ so $t(2v/3) = d$, $t = (3/2)(d/v) = (3/2)(4) = 6$ seconds. The answer is *not* 5.
14. Answer: $4 - \pi$. Connect all the centres to obtain a square of side length 2 and area 4. The shaded region is this square with 4 quarter circles removed, and has area 4 minus the area of a whole circle, namely $4 - \pi$.
15. Answer: $2\sqrt{2} - 3/2$. Suppose Alan's head is directly above the point where the ramp meets the ground, so that $x = 1.5$. Then, by a 45-45-90 right triangle, the height of the head above the ground would be $(3\sqrt{2})/2 > 2$. We need to lower his head by $(3\sqrt{2})/2 - 2$. Again, by right triangles, this means moving down the ramp a distance equal to $((3\sqrt{2})/2 - 2)\sqrt{2} = 3 - 2\sqrt{2}$, so the required x is $3/2 - (3 - 2\sqrt{2}) = 2\sqrt{2} - 3/2$.
16. Answer: She should not spin. We know Jenny has one of the 5 empty spots in front of her. Of these spots, 3 have a chocolate to the right and so if Jenny does not spin and tries the compartment to the right she has a 3/5 chance of getting a chocolate. If she does spin then her chances are 7/12. Since $5 \times 7 < 3 \times 12$, $7/12 < 3/5$ and so Jenny is better off not spinning.
17. Answer: 6. Let the big radius be a and the small one b. Drop a perpendicular from O to the line AB that meets AB at C say. The length of AB is twice that of AC, which by Pythagoras is $\sqrt{a^2 - b^2} = 3$ using $\pi a^2 - \pi b^2 = 9\pi$.
18. Answer: 2/3 m. The required distance is 1/2 + 1/8 + 1/32 +… If we call this sum S then $S/2 = 1/4 + 1/16 + 1/64$ +…, so $S + S/2$ equals 1/2 + 1/4 + 1/8 + 1/16 + 1/32 + 1/64 +… = 1, giving $S = 2/3$.