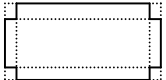# UBC Grade 6–7 Workshop Problems, 2008–2009

1. A rectangular piece of paper has an area of 50 cm^2. The paper can be folded in half into a square with a single fold. What is the perimeter of the paper?

2. Gary goes to ToysRUs to buy a limited-edition action figure, which has a sticker price of $60. While in the checkout line, he sees a sign that reads "buy any three action figures and get 50% off the cheapest one." So, Gary rushes back to the aisle and picks up two more action figures, including one with a sticker price of $110. Gary is so excited he forgets the price of the third action figure, but he remembers that all the prices were different and that he paid a pre-tax total of $190. What was the sticker price of the third action figure Gary bought?

3. Starting today, Martha bakes one pie a day for 6 straight days. Each day she offers 1/3 of the delicious but fattening pie to her father, 1/4 to her mother, 1/6 to her older sister, and 1/8 to her younger brother. Every other day, starting on the second day, Martha's mother declines her share and returns it to Martha. Her sister also returns her share to Martha every third day, starting on the third day, while the male members of Martha's family eat their share every day. Martha eats all the pie that is not eaten by another member of her family. Who ends up eating more pie, Martha or her father? Does your answer change if Martha bakes pies for 30 straight days instead of 6 days?

4. In a bakery, several different types of dessert are sold to the customers. Each dessert has sponge cake, cheesecake, mousse, or pudding as its base. A customer must add exactly two different toppings chosen from strawberries, blueberries, chocolate sprinkles, and cookie wafers, but cannot choose both fruits. How many different types of dessert does the bakery sell (the order of the toppings doesn't matter)?

5. In a class of 40 students, 60% are boys and 40% are girls. As their math activity one day, they decide to measure their weights and compute some averages. They find that the average weight of all the boys is 40 kg, and by coincidence that the average weight of *all* members of the class, including the 80-kg teacher, also equals 40 kg. What is the average weight of the girls in the class?

6. A rectangular piece of cardboard has four small squares removed from its corners and is then folded to make an open-top box that has volume 64 cm^3. The width of this box is half the length, and the height is half the width. Find the dimensions of the original piece of cardboard.

7. Rohin helps his dad do laundry every weekend. One Saturday they do three loads, the first with white items, the second with black items, and the third with coloured items. Each load contains both towels and clothes. The first load weighs 6 kg and is 1/3 towels (by weight) and 2/3 clothes. The second load weighs 9 kg and is 2/3 towels and 1/3 clothes. The final load is 1/4 towels and 3/4 clothes. Rohin has a strong sense of equality and wants to be sure that the total weight of towels washed equals the total weight of clothes washed. How many kilograms of towels and clothes should Rohin tell his dad to wash in the final load?

8. Shawn bought a bicycle and sold it to Rachel for 5/6 the price he paid. Rachel sold it to Raelene for 1/5 less than she paid. Raelene sold it to Rick for 75% of the price she paid. If Shawn paid $54.66, how much did Rick pay for the bicycle? Do you notice something special about your answer? Explain.

9. A square piece of paper is folded in half along a diagonal into a triangle. This triangle is itself folded in half into a smaller triangle, which is folded again, and the process is repeated until the paper has been folded a total of 10 times. If the longest side of the final triangle has length 1 cm, what was the length of the side of the original square?

10. Zach and Reana both like baking cupcakes. They have access to only one oven and unfortunately can't cooperate to make a batch of cupcakes together. For every batch, they decide which one of them gets to bake the cupcakes by flipping a coin. The oven is small and each batch yields only four cupcakes. The winner eats one cupcake, and the loser must buy one cupcake from the winner for $1. The other two cupcakes are sold to other friends for $0.50 each. After a number of batches have been baked, Zach has baked three batches and Reana has earned a net amount of $5. How many batches has Reana baked?

11. A three-digit number has the following properties: (1) the sum of all the digits is even; (2) each of the three digits is a prime number; (3) the digits are all different. How many such numbers are there? Remember: A number is *prime* if it has *exactly* two divisors; the first three prime numbers are 2, 3, and 5.

12. There are 26 cards, each of which has a letter on one side and a number on the other side. You are told that the cards satisfy the rule: "if there is a vowel on one side, then there is an even number on the other side." You can see one side of each card. Eight cards have the numbers 22, 23, 31, 33, 44, 45, 47, and 48. The other 18 cards have the letters A, B, C,…, R. What is the smallest number of cards you must turn over to check whether the rule is satisfied?

13. A hotel owner is very superstitious and avoids room numbers that include the string "13." For example, he doesn't number any room 213 or 1395. The first room is numbered 101, and the other rooms are given increasing consecutive numbers, with the unlucky numbers containing 13 skipped. The last room has is numbered 2009. How many rooms are in the hotel?

14. A mathematically inclined meteorologist has discovered that rainfall in Raintopia follows an amazingly regular pattern each month. On dates that are divisible by 2 but not 3 the rainfall is 30 mm. On dates that are divisible by 3 the rainfall is 40 mm. (Raintopia gets a *lot* of rain!) On dates that are not divisible by 2 or 3, it does not rain in Raintopia. What are the greatest and least amounts of total rainfall in Raintopia in a period of 11 consecutive days in October?

15. There are two bags, with contents as shown at the right. Mohammad takes one or two shapes from Bag A and then one or two shapes from Bag B. How many possible combinations of shapes can Mohammad have? The order in which the shapes are selected does not matter. For example, choosing a square from Bag A and a circle from Bag B is considered the same as choosing a circle from Bag A and a square from Bag B.

Bag A   Bag B
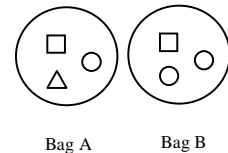
16. You have a standard deck of 52 cards. Each card has a certain value, and there are four cards of each value. An ace has value 1, a jack has value 11, a queen has value 12, and a king has value 13. The other 36 cards have value 2, 3, 4, 5, 6, 7, 8, 9, or 10. How many different ways are there of picking two cards from the deck so that the sum of the values is less than 10? Note that the order of the two cards doesn't matter. For example, picking the 3 of diamonds first and the 2 of hearts second is the same as picking the 2 of hearts first and the 3 of diamonds second.

17. Eight marbles all look alike, but one of them is actually slightly heavier than the other seven, which all have the same weight. Using a balance scale, which lets you determine which of two chosen collections of marbles, if any, has greater total weight, explain how you can find the heavier marble with just three weighings. Can you find the heavier marble with just two weighings?

SOLUTIONS

Note: These concise solutions are meant for workshop leaders and teachers. Presentations to Grade 6/7 students should include additional detail and motivation. The solutions outlined here are considered appropriate for school students at this level; alternate solutions are often possible.

1. Answer: 30 cm. Draw a picture. The square has area 25 cm^2, which is half the original area. Since 5 x 5 = 25, the square has side length 5 cm. So, the rectangle is 5 cm x 10 cm, with perimeter 2 x (5 + 10) = 30 cm.

2. Answer: $40. The $110 action figure can't be the cheapest one, so it is not the one that was discounted. That leaves $80 = $190 - $110 as the actual pretax amount paid for the other two action figures. If the third action figure had a sticker price greater than $60 then the first action figure was the discounted one and would have cost $30, so these two action figures would have together cost more than 30 + 60 = $90. So, the third action figure had a sticker price less than $60 (it can't be exactly $60 since all the prices are different), and it was the discounted one. The discounted price of the third action figure was 80 – 60 = $20, so the sticker price was $40.

3. Answers: Martha's father; no. During the 6 days, Martha's dad eats 1/3 x 6 = 2 pies in total. Her mother eats 1/4 x 3 = 3/4 pies in total. Her older sister eats 1/6 x 4 = 2/3 pies in total. Her younger brother eats 1/8 x 6 = 3/4 pies in total. So, in these 6 days, Martha eats a total of 6 – 2 – 3/4 – 2/3 – 3/4 = 4 – 3/2 – 2/3 = 4 – 9/6 – 4/6 = 11/6 pies, which is less than her father ate. If Martha bakes pies for 30 days instead, then since the pattern of pie eating repeats every 6 days and 30 is a multiple of 6, the answer does not change.

4. Answer: 20. There are 4 possible bases. There are 5 choices of pairs of toppings other than the excluded one (list them), so there are 4 x 5 = 20 types of dessert.

5. Answer: 37.5 kg. There are 0.6 x 40 = 24 boys and 0.4 x 40 = 16 girls in the class. Since the average of some quantities is the sum of those quantities divided by the number of quantities, the sum is the average multiplied by the number. So, the sum of the weights of the boys is 40 x 24 = 960 kg. Now, for the average of all the members of the class to be 40 also, we need the total weight of the members of the class to equal 40 x 41 = 1640 kg. Since the teacher weighs 80 kg, the total weight of the girls is 1640 – 960 – 80 = 600 kg. The average weight of the girls is 600/16 = 75/2 = 37.5 kg.

6. Answer: 8 cm x 12 cm. By trial and error, or by noting that since the length is twice the width and the height is half the width the volume equals the cube of the width and 4^3 = 64, the dimensions of the box are 2 cm x 4 cm x 8 cm. The four small removed squares are 2 cm x 2 cm, and the dimensions of the original cardboard are 2 + 4 + 2 = 8 cm and 2 + 8 + 2 = 12 cm.

7. Answer: 2 kg total (0.5 kg towels, 1.5 kg clothes). The first load has 6 x 1/3 = 2 kg of towels and 6 x 2/3 = 4 kg of clothes. The second load has 9 x 2/3 = 6 kg of towels and 9 x 1/3 = 3 kg of clothes. In total, the first two loads have 2 + 6 = 8 kg of towels and 4 + 3 = 7 kg of clothes. We need to have one more kg of towels than clothes in the third load. If we were to wash 1 kg total in the final load, we would have washed 3/4 – 1/4 = 1/2 more kg of towels than clothes. So, we need to wash 2 kg total.

8. Answer: $27.33. Calculating all the prices directly leads to the answer. However, we notice that the answer is half the original price paid, and can explain that by noting that Raelene paid 4/5 what Rachel paid and Rick paid 3/4 what Rachel paid, so, multiplying the fractions, Rick paid 5/6 x 4/5 x 3/4 = 1/2 what Shawn paid.

9. Answer: 16 cm. Draw a picture or use a prop. Note that after the first two folds, we have a triangle whose longest side has length equal to that of the original square. After another two folds, the triangle's longest side is half as long, and each pair of subsequent folds introduces another factor of 1/2. So, after 10 folds, i.e. 4 pairs of folds after the first two folds, the longest side of the triangle is 1/2 x 1/2 x 1/2 x 1/2 = 1/16 the length of the side of the square.

10. Answer: 4. For each batch, the winner makes a total of $2 and the loser loses $1. Since Zach has baked three batches, Reanna has lost $3 during these three batches. In order to have earned a net amount of $5, Reanna must have earned a total of $8, which means she has baked four batches.

11. Answer: 18. The digits must be 2, 3, 5, or 7, since these are the only primes less than 10. Since the sum of three odd numbers is odd and all but one of these digits is odd, one of the digits must be 2, with the other two digits being two of 3, 5, and 7. If the 2 appears as the 1s digit, there are 6 numbers, namely 352, 532, 372, 732, 572, and 752. Similarly, there are 6 numbers if 2 appears as the 10s digit and another 6 if it appears at the 100s digit.

12. Answer: 9 cards. We need to turn over the 4 cards showing a vowel, namely the cards showing A, E, I, and O, to verify that the reverse has an even number. We *don't* need to turn over the cards showing consonants since it does not matter what type of number is on the reverse of these cards. Also, we need to turn over any card showing an *odd* number, to check that there is not a vowel on the other side, since in if there were the rule would be in violation. There are 5 such cards. We don't need to turn over a card showing an even number.

13. Answer: 1771. Count the number of numbers between 101 and 2009 that *do* include a string 13, and subtract this number from 1909, which is the total number of numbers between 101 and 2009. Systematically list the numbers containing 12. There are no 2-digit numbers since the smallest room number is 101. There are 9 3-digit numbers with 13 as the last two digits, namely 113, 213,…, 913. There are 10 3-digit numbers with 13 as the first two digits, namely 130, 131,…, 139. Note that there is no overlap between these two lists. There are 10 4-digit numbers ending in 13, namely 1013, 1113,…, 1913. There are also 10 4-digit numbers with 13 as the middle two digits, namely 1130, 1131,…, 1139, and these do not overlap with the first list of 4-digit numbers. Finally, there are 100 4-digit numbers that start with 13, namely 1300, 1301,…, 1399, but one of these, 1313, has already been counted. So, the number of numbers that include the string 13 is 9 + 10 + 10 + 10 + 99 = 138, and the number that don't is 1909 – 138 = 1771.

14. Answer: greatest 280 mm, least 240 mm. In a stretch of 11 consecutive numbers, 3 of them are divisible by 3 if the first number is 1 more than a multiple of 3, and 4 such numbers are otherwise. Also, in such a stretch 3 numbers are divisible by 2 but not 3 if the first number is 3 or 5 more than a multiple of 6, and 4 such numbers are otherwise. (Both of these sets of statements require explanation; the observation that there are exactly 2 multiples of 6 among any 11 consecutive integers is useful for the second set.) We can maximize the total 11-day rainfall at 4 x 40 + 4 x 30 = 280 mm if we can make the first date not 1 more than a multiple of 3 and not 3 or 5 more than a multiple of 6, which we can for example be letting the first date be 2. We cannot achieve the *possible* minimum value of 3 x 40 + 3 x 30 = 210 mm since there is no number that is both 1 more than a multiple of 3 and 3 or 5 more than a multiple of 6. The actual minimum is 3 x 40 + 4 x 30 = 240 mm, obtained by letting the first date be 1 more than a multiple of 3, for example 1.

15. Answer: 16. Suppose one shape is chosen from each bag. Then we have two shapes in total, and there are 5 possible combinations; the only one of the 6 a priori possibilities — 3 in which both shapes are the same and 3 in which they are different — that is not allowed is two triangles. If one shape is chosen from one bag and two from the other, there are a priori 10 possibilities — 3 in which both shapes are the same, 6 in which one shape appears twice together with a sole different shape, and 1 in which all three shapes appear exactly once — and of these 10 possibilities the only 4 that don't actually occur are three square, three triangles, and two triangles together with some other shape. So, there are 10 – 4 = 6 possible combinations with a total of three shapes. Finally, if two shapes are chosen from each bag, then we can have: three circles together with a square or triangle, two circles and two squares, two circles together with a triangle and square, or two squares together with a circle and triangle, a total of 5 possible combinations. The total number is 5 + 6 + 5 = 16.

16. Answer: 280. Note first that none of the cards can have a value of 9 or larger, or else the sum would be at least 10, so we can ignore 9s, 10s, jacks, queens, and kings. If the first card picked has value 1, there are 4 choices for it, and the second card must have value between 1 and 8. Since one 1 has already been chosen, there are 3 + 4 x 7 = 31 choices for the second card. If the first card picked has value 2, the second card must have value between 1 and 7, and there are 3 + 4 x 6 = 27 choices. If the first card is a 3, there are 23 choices for the second, and if the first card is a 4 there are 19 choices. However, the pattern does not continue since if the first card is a 5, the second must have value between 1 and 4, and there are 16 choices. There are 12 choices when the first card is a 6, 8 if it is a 7, and 4 if it is an 8. Adding gives 4 x (31 + 27 + 23 + 19 + 16 + 12 + 8 + 4) = 560 *ordered* pairs, and dividing by two since the order does not matter gives the final answer. Alternate solution (sketch): There are 4 x 6 = 24 choices in which the same value appears (two aces, two 2s, two 3s, or two 4s). There are, as above, (7 + 6 + 5 + 4 + 4 + 3 + 2 + 1)/2 = 16 unordered pairs of distinct numbers whose sum is less than 10. The final count is 24 + 4 x 4 x 16 = 280.

17. Answer: Yes. For the first part: put 4 marbles on one side of the scale and the other 4 on the other side. The side with the heavier marble will be heavier in total, so we will know which set of 4 marbles contains the heavier one. Repeat, breaking the 4 marbles including the heavier one into two sets of 2 marbles to determine a pair of marbles one of which is the heavier one, and then in a final third weighing separate these 2 marbles to determine the heavier one. In fact two weighings *are* enough: Leave two marbles alone and break up the other 6 up into two sets of 3, placing each set on one side of the scale. If the heavier marble is among these 6, one of the sets of 3 will be heavier. Leave one of these heavier 3 marbles along and weigh the other 2. If these 2 marbles have the same weight, then the marble that was not weighed in the second weighing is the heavier one; if not, the heavier marble will have been determined. If the two sets of 3 marbles have the same weight, the heavier marble is one of the 2 that were not originally weighed, and the heavier one of these can be determined with a second weighing.