Mathematics 414, Problem Set #5
(due by 1:00, October 13)

Problem 1. Let n be a positive integer. Using school level methods, show that $1+1/2^2+1/3^2+\cdots+1/n^2 < 1.7$. (Euler showed that the infinite sum $1+1/2^2+1/3^2+\cdots$ is equal to $\pi^2/6$, but I know of no high school level proof.) Hint: Approximate the "tail" (the stuff from $1/(k+1)^2$ on, for suitable k), by something easily summed.

Problem 2. What is the sum of the *real* roots of $x^4 - 2x^3 + x^2 - 2x + 1$? Hint: Divide by x^2 and let $u = x + 1/x$.

Problem 3. There are 6678 powers of 2 between 1 and 10^{2010}. Roughly how many have a decimal representation that begins with 1? Do not attempt to find an exact answer: informed speculation is good enough. (One might guess around 6678/9, but I think such a guess is unreasonable, and indeed it is wildly wrong.)

Problem 4. Make up and solve a workshop problem that involves counting. The problem should be ready to type "as is."

Problem 5. Make up and solve a workshop problem, something enlightening about calculator computations, or something that involves series. The problem should be ready to type "as is."

Assignment: Continue to read carefully the 2009–2010 UBC workshop problems and solutions. Write out for yourself solutions to the problems, solutions that when necessary go beyond or improve on the solution sketches provided. The midterm (scheduled for November 5) will be closely related to these workshop problems.