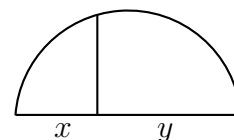


Mathematics 414, Problem Set #4
(due by 1:00, October 6)

Problem 1. Use the semicircle below to give a geometric argument for the two-variable Arithmetic Mean — Geometric Mean Inequality (if x and y are positive, then $(x + y)/2 \geq \sqrt{xy}$, with equality precisely if $x = y$).



Problem 2. (a) Define the sequence A_0, A_1, A_2 , and so on by $A_0 = A_1 = 1$, and $A_n = 2A_{n-1} + A_{n-2}$ for $n \geq 2$. Let $x = 1/3$. Calculate

$$A_0 + A_1x + A_2x^2 + A_3x^3 + \cdots + A_nx^n + \cdots .$$

Manipulate infinite sums freely, ignoring issues of convergence.

(b) For $x = 1/3$, find $\sum_{n=1}^{\infty} nA_nx^n$.

Problem 3. Solve *exactly one* of the following two problems:

(i) Find the slope of the tangent line to the curve $xy = 4$ at the point $(1, 4)$ in two ways, neither of which involves “calculus.” One way I can think of uses routine algebra, another uses a transformation.

(ii) Beth has a biased loonie that lands heads with probability $p \neq 0$, and tails with probability $1 - p$. Alicia tosses the coin repeatedly, and keeps a running count of heads and tails. If the number of heads is *ever* greater than the number of tails, Alicia wins the game (and the coin). What is the probability that Alicia wins the game?

Problem 4. Make up and solve a grade 6/7 workshop problem with geometric content. The problem and solution needs to be ready for your secretary (me) to type “as is.” Some interesting mathematical point should be involved.

Problem 5. Make up and solve a grade 11/12 workshop problem. The inspiration (possibly indirect) should come from an explicitly identified Euclid problem *beyond* Euclid 1–4. The problem should be some distance away from the inspiration source, for the sake of novelty, and to avoid copyright issues.

Assignment: Continue to read carefully the 2009–2010 UBC workshop problems and solutions. Write out for yourself solutions to the problems, solutions that when necessary go beyond or improve on the solution sketches provided. The midterm will be closely related to these workshop problems.