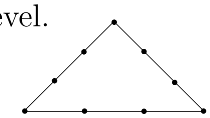
Mathematics 414, Problem Set #3

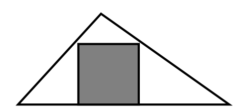
(due by 1:00, September 29)

Problem 1. A conical cup was filled to the rim with brandy. Xavier had some. When he was finished, the brandy level was down one-quarter of an inch. Yolande then drank, leaving the bottom two inches for Zoë. It turns out that Xavier and Zoë had exactly the same amount of brandy. How tall is the cup? (You may use grade 11/12 techniques, but preferably should avoid the standard volume of cone formula.)

Problem 2. How many triangles have their vertices at 3 of the 9 points in the figure below? A triangle has been drawn to make the location of the points clearer—and it is one of the triangles we need to count. The solution *cannot* (explicitly) use the binomial coefficients $\binom{n}{r}$. Use only tools available at the grade 6–7 level.



Problem 3. A square has area 1. (a) What is the smallest possible area among all triangles such that (i) one side of the square lies on a side of the triangle and (ii) the triangle contains the square? (b) Among all such triangles of smallest area, what is the smallest possible perimeter?



Problem 4. Design a geometric or combinatorial (counting) problem, and write out a solution (or solutions, when appropriate). Aim at the level of the Euclid contest, problems 5–7.

Problem 5. Design *two* problems for Math Challengers (MC), supplying answers, but *not* solutions. The answer to an MC problem is almost always a single number. Traditionally, MC has not allowed the use of a calculator (that will change). No student past grade 9 can participate. The contestants are mathematically pretty good, but have limited time for each question, roughly from 1 to 5 minutes. Since this is a contest, clarity of wording is very important (we don't want to be sued by a disappointed competitor). One of your problems should be fairly easy, the other not so easy, and the problems should be in different areas.

Assignment: Continue to read carefully the 2009–2010 UBC workshop problems and solutions. I would hope that the grade 6–7 problems have been absorbed, so one should move on to the grade 8–10.