Math 100:V02 - WORKSHEET 7 THE CHAIN RULE

1. THE CHAIN RULE

- (1) We know d/dy sin y = cos y.
 (a) Expand sin(y + h) to linear order in h. Write down the linear approximation to sin y about y = a.
 - (b) Now let $F(x) = \sin(3x)$. Expand F(x+h) to linear order in h. What is the derivative of $\sin 3x$?

Fact. $(f(g(x)))' = f'(g(x))g'(x) \text{ or } \frac{\mathrm{d}}{\mathrm{d}x}(f(g(x))) = \frac{\mathrm{d}f}{\mathrm{d}g} \cdot \frac{\mathrm{d}g}{\mathrm{d}x}.$

(2) Write each function as a composition and differentiate (a) e^{3x}

(b) $\sqrt{2x+1}$

- (c) (Final, 2015) $\sin(x^2)$
- (d) $(7x + \cos x)^n$.
- (3) (Final, 2012) Let $f(x) = g(2\sin x)$ where $g'(\sqrt{2}) = \sqrt{2}$. Find $f'(\frac{\pi}{4})$.

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(4) Differentiate

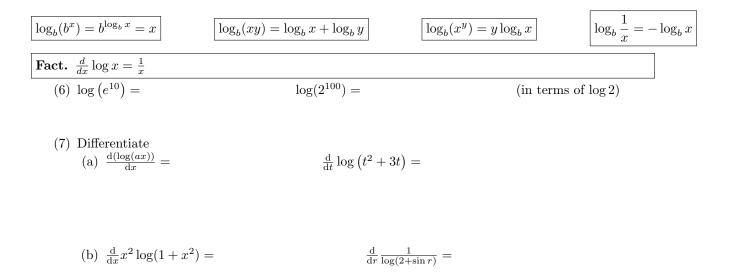
(a) $7x + \cos(x^n)$

(b) $e^{\sqrt{\cos x}}$

(c) (Final 2012) $e^{(\sin x)^2}$

(5) Suppose f, g are differentiable functions with $f(g(x)) = x^3$. Suppose that f'(g(4)) = 5. Find g'(4).

2. Differentiating logarithms



(8) (Logarithmic differentiation) Use $\log(fg) = \log f + \log g$ to differentiate $y = (x^2 + 1) \cdot \sin x \cdot \frac{1}{\sqrt{x^3 + 3}} \cdot e^{\cos x}$.

(9) Differentiate using
$$f' = f \times (\log f)'$$

(a) $\star x^n$

(b) x^{x}

(c) $(\log x)^{\cos x}$

(d) (Final, 2014) Let $y = x^{\log x}$. Find $\frac{dy}{dx}$ in terms of x only.

3. More problems

(10) Let $f(x) = g(x)^{h(x)}$. Find a formula for f' in terms of g' and h'.

- (11) Let $f(\theta) = \sin^2 \theta + \cos^2 \theta$. Find $\frac{df}{d\theta}$ without using trigonometric identities. Evaluate f(0) and conclude that $\sin^2 \theta + \cos^2 \theta = 1$ for all θ .
- (12) ("Inverse function rule") suppose f(g(x)) = x for all x. (a) Show that $f'(g(x)) = \frac{1}{g'(x)}$.

(b) Suppose $g(x) = e^x$, $f(y) = \log y$. Show that f(g(x)) = x and conclude that $(\log y)' = \frac{1}{y}$.

(c) Suppose $g(\theta) = \sin \theta$, $f(x) = \arcsin x$ so that $f(g(\theta)) = \theta$. Show that $f'(x) = \frac{1}{\sqrt{1-x^2}}$.

(13) (Final, 2015) Let $xy^2 + x^2y = 2$. Find $\frac{dy}{dx}$ at the point (1, 1).