Math 100:V02 – WORKSHEET 6
EXPONENTIAL AND TRIG FUNCTIONS

1. Review: Arithmetic of derivatives

Fact. $(af+bg)' = af' + bg'$, $\quad (fg)' = f'g + fg'$, $\quad \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$
$\frac{d}{dx}x^n = nx^{n-1}$, $\frac{d}{dx}e^x = e^x$

(1) Differentiate
   (a) (Final, 2016) $g(x) = x^2 e^x$ (and then also $x^a e^x$)

   (b) (Final, 2016) $h(x) = \frac{x^2+3}{2x-1}$

(2) Let $f(x) = \frac{x}{\sqrt{x}+A}$. Given that $f'(4) = \frac{3}{16}$, give a quadratic equation for A.

(3) Suppose that $f(1) = 1$, $g(1) = 2$, $f'(1) = 3$, $g'(1) = 4$.
   (a) What are the linear approximations to f and g at $x = 1$? Use them to find the linear approximation to fg at $x = 1$.

   (b) Find $(fg)'(1)$ and $\left(\frac{f}{g}\right)'(1)$.

(4) Evaluate
   (a) $(x \cdot x)'$ and $(x') \cdot (x')$. What did we learn?

   (b) $\left(\frac{x}{x}\right)'$ and $\frac{(x')}{(x')}$. What did we learn?

Date: 25/1/2024, Worksheet by Lior Silberman. This instructional material is excluded from the terms of UBC Policy 81.

2. Exponentials

(5) Simplify
   (a) $\left(e^5\right)^3$, $\left(2^{1/3}\right)^{12}$, 7^{3-5}.

   (b) $\log(10e^5)$, $\log(3^7)$.

(6) Differentiate:
   (a) 10^x

   (b) $\frac{5\cdot 10^x + x^2}{3^x+1}$

3. Trigonometric functions

Fact. *When x is measured in* ***radians****, we have* $(\sin x)' = \cos x$, $(\cos x)' = -\sin x$

(7) (Special values) What is $\sin\frac{\pi}{3}$? What is $\cos\frac{5\pi}{2}$?

(8) Derivatives of trig functions
   (a) Interpret $\lim_{h\to 0}\frac{\sin h}{h}$ as a derivative and find its value.

   (b) Differentiate $\tan\theta = \frac{\sin\theta}{\cos\theta}$.

(9) What is the equation of the line tangent the graph $y = T\sin x + \cos x$ at the point where $x = \frac{\pi}{4}$?