Math 100:V02 – WORKSHEET 1
EXPRESSIONS AND ASYMPTOTICS

1. The ladder of functions

(1) Classify the following functions into *power laws / power functions* and *exponentials*: x^3, πx^{102}, e^{2x}, $c\sqrt{x}$, $-\frac{8}{x}$, 7^x, $8 \cdot 2^x$, $-\frac{1}{\sqrt{3}} \cdot \frac{1}{2^x}$, $\frac{9}{x^{7/2}}$, x^e, π^x, $\frac{A}{x^b}$.

(2) Order the following functions from small to large *asymptotically as* $x \to \infty$*:*
   (a) 1, $\sqrt{x}$, $x^{-1/2}$,$x^{1/3}$, e^x, $x^{-1/3}$, $10^6 x^{2024}$, e^{-x}, e^{x^2}, $\frac{2024}{x^{100}}$, 5^x, x.

   (b) Extra: add in $\log x$, $e^{\sqrt{x}}$, $(\log x)^2$, $\log\log x$, $\frac{1}{\log x}$.

   (c) Repeat (a), this time as $x \to 0^+$.

2. Asymptotics: simple expressions

(3) How does the each expression behave when x is large? small? what is x is large but negative? Sketch a plot
   (a) $1 - x^2 + x^4$ (“Mexican hat potential”)

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(b) $ax^3 - bx^5$ $(a, b > 0)$

(c) $e^x - x^4$

(d) Wages in some country grow at 2% a year (so the wage of a typical worker has the form $A \cdot (1.02)^t$ where t is measured in years and A is the wage today). The cost of healthcare grows at 4% a year (so the healthcare costs of a typical worker have the form $B \cdot (1.04)^t$ where B is the cost today). Suppose that today's workers can afford their healthcare (A is much bigger than B). Will that be always true? Why or why not?

(e) Three strains of a contagion are spreading in a population, spreading at rates 1.05, 1.1, and 0.98 respectively. The total number of cases at time t behaves like

$$A \cdot 1.05^t + B \cdot 1.1^t + C \cdot 0.98^t \, .$$

(A, B, C are constants). Which strain dominates eventually? What would the number of infected people look like?

(4) The (attractive) interaction between two hadrons (say protons) due to the strong nuclear force can be modeled by the *Yukawa potential* $V_{\mathrm{Y}}(r) = -g^2 \frac{e^{-\alpha m r}}{r}$ where r is the separation between the particles, and g, α, m are positive constants. The elecctrical repulsion between two protons is described by the Columb potential $V_{\mathrm{C}}(r) = kq^2 \frac{1}{r}$ where k, q are also positive constants. Which interaction will dominate for large distances? Will the net interaction be attractive or repulsive? Note that g^2 is much larger than kq^2.

3. Asymptotics of complicated expressions

(5) Describe the following expressions in words

(a) $e^{|x-5|^3}$

(b) $\frac{1+x}{1+2x-x^2}$

(c) $\frac{e^x+A\sin x}{e^x-x^2}$

(d) $\frac{Ae^{rt}+Be^{-st}}{t+t^2}$ where $r, s > 0$ and $A, B \neq 0$.

(6) For each of the functions in (a),(b),(c),(d) determine its asymptotics near 0 and near $+\infty$.

(a)

(b)

(c)

(d)