Math 223: Linear Algebra
Winter Semester, 2022

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Course Website	`http://www.math.ubc.ca/~lior/teaching/2122/223_W22/`
Contact me at	[MATX 1112 — 604-827-3031] – lior@math.ubc.ca
My Website	`https://www.math.ubc.ca/~lior/`
Class	MWF 10:00-10:50 at MATH 104 (initially on Zoom)
Office Hours	TBA in-person and on Zoom
Textbook	None required; see below.
Course Prerequisites	At least one of: (a) MATH 121 or (b) a score of 68% or higher in MATH 101, MATH 103, MATH 105, or SCIE 001.

About the course

We will explore linearity, one of the most basic mathematical phenomena, and Linear algebra, a basic mathematical language which is used in every part of mathematics and generally within science to describe the world around us. For example, it is the most natural language to express quantum mechanics, is basic to studying signal processing, and underlies calculations in computer graphics. Beyond students interested in becoming mathematicians, this course is essential for students interested in theoretical physics and computer science. We will emphasize mathematical ways of thought, including understanding definitions and writing rigorous proofs.

> **This course is aimed at excellent students (typically honours students, though anyone may enroll). It is more abstract and covers more material than MATH 152 and MATH 221.**

The textbook for this course is *[2], Linear algebra* by Friedberg, Insel and Spence. It will serve as a reference and as a source of practice problems. That said, the material is standard and is covered by many textbooks, including several that can be freely downloaded through the UBC library. Purely as examples note [4] (inexpensive with an emphasis on worked problems), and [1, 3] (abstract treatments; can be downloaded from SpringerLink by following the links while on the UBC VPN).

There will be three parallel themes through the course: computation, abstract linear algebra, and general mathematical thinking. Exams will consist roughly of 30-40% computation and 60%-70% abstract material.

New this year

There will be several interrelated changes to the course this year.

1. Layered learning goals: the learning goals have been redesigned to clarify the expectations for every achievement (from passing up to A+).
2. Layered homework: the homework will be divided accordingly.

3. Mastery of basic skills: Certain very basic skills and knowledge (primarily calculation also the basic definitions and their consequences) are crucial in this course, and students will be expected to *master* them, as follows:
   (a) There will be essentially no part marks in midterm and final exam questions related to these topics.
   (b) Passing the course will require scoring at least 80% on this material in the midterm. Unlimited retakes of the midterm will be offered.
4. *Recitation sections*: an (optional) weekly meeting with a TA.

Teaching and learning

Significant prerequisites

- None really. The course is almost entirely self-contained.
- The formal calculus pre-requisites is intended to gauge the level of mathematical maturity of the students; strong students can and should ask for it to be waived. We will rarely use high-school level calculus (integrals and derivatives), and only as a source of examples.

When and where

- We will meet on MWF 10:00–10:50 at the classroom MATH 104.

  This course will be online for a portion of the term. We will start in-person meetings on a date yet to be determined by the University, not earlier than January 24th but quite possibly later. When we resume in-person activities, we will meet in our scheduled classroom space on campus, and until then lectures will take place on Zoom (link on Canvas; if you aren't registered in the course ask the instructor for the link).

- There will be office hours after class, and additional office hours at times to be determined later, including Zoom office hours.

  During the online portion of the semester, the after-class office hours will be on Zoom as well.

- An assistant will hold *recitation sections* at a time to be determined.
- There will be a Piazza discussion board accessible through Canvas. Use it to ask *and* answer questions about the course.

Learning goals

A lecture-by-lecture table of the material of will be posted to the course website. A detailed list of learning goals is attached to the end of this document, divided into four groups: *computational skills*, *"language" skills* (treating linear algebra as a language), *abstract algebra skills*, and *metamathematical skills* (such as writing rigorous proofs).

Expectations

You can expect from me:

- To come prepared for class: knowing what we want to achieve, and how we will achieve it.
- Responses to your questions and concerns: continuously in class and during my office hours, within reasonable time on Piazza and by e-mail outside class.

- Demanding homework and examinations.
- Clear explanations of what is correct in your work and what is not, and help in improving.

You are expected:

- To come prepared to class, having read relevant material and done problem sets.
  - Working on the problem sets is *absolutely essential* for learning the material. **It is extremely rare for students who skip problem sets to do well on exams.**
- Actively participate in the course: read ahead of class, think about the material, and **ask questions**.
  - Asking questions when you don't understand, or want to learn more, ensures that you get what you want out of the course. Ask me questions in class, on Piazza, by email, and during office hours. Also, ask your colleagues questions outside of class – you will both benefit from the discussion!
- To submit written work that is readable and communicates your ideas.
- To *typset* your homework.
- To master the computational material on your own.

Official Policies

General policies

- **Late or missed exams and assignments will not be accepted for credit and will be given a grade of zero**. In exceptional circumstances the missed work will be registered if you finish it and hand it in when you can.
  - Registered homework will not count when the homework average is computed. The weight of a registered midterm will be transferred to the final exam.
  - If you need to miss work / have missed work please let me know as soon as possible. Sometimes this means letting me know well in advance (example: you are scheduled to represent UBC in an athletic competition later in the term), but sometimes after the fact (example: you fall and break your arm, and write to me only after you are ready to resume schoolwork).
  - In common situations (medical / conflicting responsibilities / compassionate grounds) no documentation is required for the **first request** for a concession. Instead, the student must submit the department's *Academic Concession Form*, available at `https://www.math.ubc.ca/Ugrad/ugradForm/Student_Declaration_Academic_Concession_MATH.pdf`.
  - First requests for concessions on other grounds, and subsequent requests for concessions must include documentation.
  - Full details may be found in Senate Policy 135.
- All assertions in your written work require ***proof*** unless the problem states otherwise. Regardless of the problem's operative word ("find", "solve", "establish", "calculate", "determine" ...), you must rigorously justify your answer.
- Written work should be presented carefully, with sufficient detail in complete English sentences. A "correct sequence of formulas" will only merit partial credit.
- I may designate material (calculational material or definitions, for example) for self-study, in which case you might be responsible for learning this material before it is used in class and in problem sets.

- You may not post homework or exam problems or *any other course material* to any website or discussion forum without coordinating with the instructor first. This includes without limitation Chegg, CourseHero, math.SE, r/math, Facebook, and the like.
  - You will be asked to pledge to not post information or consult external sources for help. This is absolutely essential: the problems are difficult, but are chosen for pedagogical reasons. Evading the difficulty harms the students who do their best without such consultation.

Homework

- There will be up to twelve problem sets posted to the course website, due on Canvas at the *beginning* of class on the day shown on the schedule. I will drop the lowest score when calculating the homework grade.
  - For the sanity of the grader, homework submissions must be *typeset* (not handwritten). The instructor uses the LyX "Document processor" but any form of typesetting is acceptable. Some suggestions may be found on the course website.
  - You are encouraged to work on solving the problems together. However, each of you must write your solutions independently, in your own words. You may (and should) share your ideas but you may not share your written work.
  - You may not post homework problems or other course material to any website or discussion forum, including but not limited to Chegg, CourseHero, math.SE, r/math, and the like.
  - It is possible that only certain problems from a problem set will be selected for grading.
  - Solutions will be posted on the secure (Canvas) website.

Exams

- There will be two (in-class) midterm exams and one final exam in the usual exam period.
  - If you need special accommodations when taking written exams, please contact the Centre for Accessibility.
  - If the midterm (or final) exam conflicts with a religious observance, or if you have any other legitimate conflict, please contact me *at least two weeks ahead of time* so we can make appropriate arrangements.
- Mastery of basic material
  - In the first part of the each exam you will be asked to demonstrate *mastery* of basic masterial (calculation, basic definitions, very basic proofs). Very little part marks will be offered in this part.
  - Scoring at least 80% on this part will be required in order to pass the course. Several retests for the "basics" part of the midterms will be offered regularly during the rest of the term.

Final grade

- The final grade will be calculated as follows:

Problem sets: 20%
Midterms: $2 \times 15\%$
Final exam: 50%

UBC boilerplate

UBC provides resources to support student learning and to maintain healthy lifestyles but recognizes that sometimes crises arise and so there are additional resources to access including those for survivors of sexual violence. UBC values respect for the person and ideas of all members of the academic community. Harassment and discrimination are not tolerated nor is suppression of academic freedom. UBC provides appropriate accommodation for students with disabilities and for religious, spiritual and cultural observances. UBC values academic honesty and students are expected to acknowledge the ideas generated by others and to uphold the highest academic standards in all of their actions. Details of the policies and how to access support are available here.

Learning goals

By the end of the course, you will have:

- Basic Computational skills:
  - Vector and matrix arithmetic, including multiplication of matrices and finding inverses;
  - Recognizing and solving systems of linear equations;
  - Transforming matrices by column and row reduction;
  - Deciding whether linear maps and matrices are invertible;
  - Computing determinants by several techniques;
  - Recognizing eigenvalue problems and computing eigenvalues and corresponding eigenvectors;
- Computational skills
  - Deciding whether sets of vectors are dependent or indepdent, including computing spans and deciding membership in subspaces;
  - Finding bases of vector spaces;
  - Relating linear maps to matrices and vice versa;
  - Computing in inner product spaces, including finding orthogonal complements.
- Language skills:
  - Converting statements in mathematics and science to the language of linear algebra;
  - Using the vocabulary of linear algebra to discuss these statements.
- Basic abstract algebra skills:
  - State the definitions and Theorems of the course;
  - Prove simple statements involving the definitions;
- Abstract algebra skills:
  - Decide whether algebraic structures are vector spaces, and whether subsets are subspaces. Decide whether maps are linear transformations;
  - Construct vector spaces from other vector spaces by various means;
  - Recognize and use linear maps, their associated kernels and images;
  - Prove statements about abstract sets.
- Metamathematical skills:

- Solving problems where the method of solution has not been given in advance;
- Applying mathematical technique to convert solution ideas to rigorous solutions;
- Constructing proofs of given statements, especially in the context of linear algebra;
- Writing clear and concise proofs;
- Recognizing whether putative proofs are correct or incorrect.

References

[1] Sheldon Axler. *Linear algebra done right*. Undergraduate Texts in Mathematics. Springer, Cham, third edition, 2015. https://link.springer.com/book/10.1007/978-3-319-11080-6.

[2] Stephen H. Friedberg, Arnold J. Insel, and Lawrence E. Spence. *Linear algebra*. Prentice Hall, Inc., Upper Saddle River, NJ, third edition, 1997.

[3] Paul R. Halmos. *Finite-dimensional vector spaces*. Undergraduate Texts in Mathematics. Springer-Verlag, New York-Heidelberg, second edition, 1974. https://link.springer.com/book/10.1007/978-1-4612-6387-6.

[4] Seymour Lipschutz and Marc Lipson. *Schaum's Outline of Linear Algebra*. Schaum's Outlines. McGraw-Hill, sixth edition, 2017.