Math 100 – WORKSHEET 20
L'HÔPITAL'S RULE

Theorem. *Let f, g be diff. near $x = a$. Suppose* $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$ *while* $\lim_{x\to a} \frac{f'(x)}{g'(x)} = L$. *Then* $\lim_{x\to a} \frac{f(x)}{g(x)}$ *exists and equals L.*

This also works if $\lim \frac{f'(x)}{g'(x)}$ exists in the extended sense ($L = +\infty$ or $L = -\infty$), if $\lim_{x\to a} f(x), \lim_{x\to a} g(x)$ are both infinite in the extended sense rather than zero, or if we take $\lim_{x\to\infty}$ (that is "$a = \infty$")

(1) Evaluate $\lim_{x\to 1} \frac{\log x}{x-1}$.

(2) (Final, 2014) Evaluate $\lim_{x\to 0} \frac{\cos x - e^{x^2}}{x^2}$.

(3) Do (2) using a 2nd-order Taylor expansion.

(4) (Final, 2015) Evaluate $\lim_{x\to 0} \frac{\log(1+x) - \sin x}{x^2}$.

(5) Given that $f(2) = 5$, $g(2) = 3$, $f'(2) = 7$ and $g'(2) = 4$ find $\lim_{x\to 3} \frac{f(2x-4) - g(x-1) - 2}{g(x^2-7) - 3}$.

(6) Evaluate $\lim_{x\to 0^+} \frac{e^x}{x}$.

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(7) Evaluate $\lim_{x\to\infty} x^2 e^{-x}$.

(8) Evaluate $\lim_{x\to 0^+} x \log x$.

(9) Evaluate $\lim_{x\to 0}(2x+1)^{1/\sin x}$.

(10) Evaluate $\lim_{x\to\infty} x^n e^{-x}$.

(11) Suppose $a > 0$. Evaluate $\lim_{x\to\infty} x^{-a} \log x$.