1. Intro; Limits (9/9/2021)

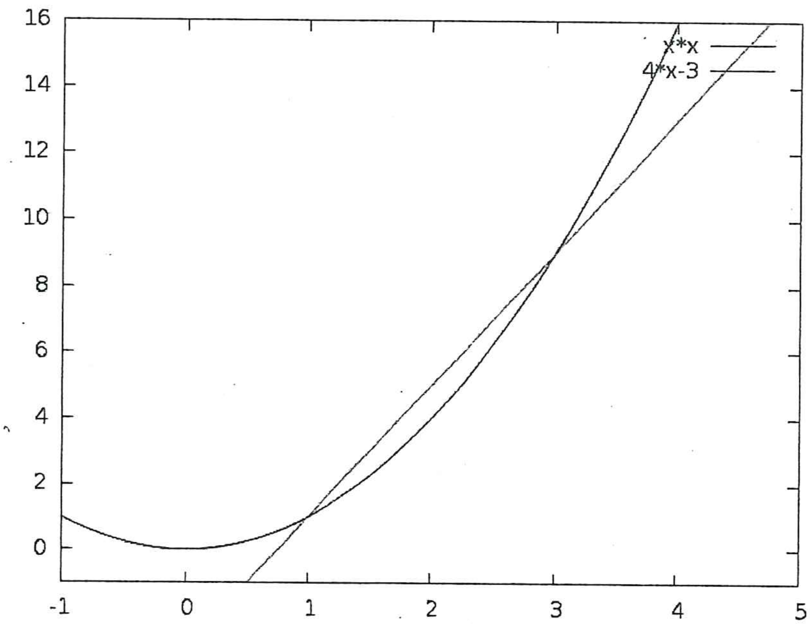
Today's Goals.

(1) Overview of the course
(2) Learning methods
(3) About me
(4) Limits: motivation and first examples

First idea: limiting processes

Math 100 – WORKSHEET 1
LIMITS

1. The slope of a graph



(1) Find the slope of the line through points $P(1,1)$ and $Q(x,x^2)$ where:

(a) $x=3$

If $Q=(3,9)$, slope $= \frac{\Delta y}{\Delta x} = \frac{9-1}{3-1} = \frac{8}{2} = 4$

(b) $x=1.1$

slope $= \frac{\Delta y}{\Delta x} = \frac{(1.1)^2-1}{1.1-1} = \frac{1.21-1}{0.1} = 2.1$

(c) $x=1.01$

$\frac{\Delta y}{\Delta x} = \frac{(1.01)^2-1}{1.01-1} = \frac{1.0201-1}{0.01} = 2.01$

(d) $x=1.001$

$\frac{(1.001)^2-1}{1.001-1} = \frac{0.002001}{0.001} = 2.001$

What is the slope of the line tangent to the curve at $P(1,1)$? What is its equation?

slope $=2$, line: $y-1=2(x-1) \iff y=2x-1$

Date: 9/9/2021, Worksheet by Lior Silberman. This instructional material is excluded from the terms of UBC Policy 81.

Again but for <u>any</u> displacement:

Say $Q(1+h, (1+h)^2)$

"name" of the displacement

<u>slope:</u> $$\frac{(1+h)^2-1}{(1+h)-1} = \frac{1+2h+h^2-1}{(1+h)-h} = \frac{2h+h^2}{h} \underset{\text{if } h\neq 0}{=} 2+h \xrightarrow[h\to 0]{} 2$$

(2) Evaluate $f(x) = \frac{x-3}{x^2-x-6}$ at $x = 2.9, 2.99, 2.999, 3.1, 3.01, 3.001$. What is $\lim_{x\to 3} f(x)$?

(3) Evaluate

(a) $\lim_{x\to 1} \sin(\pi x)$ $= \sin(\pi \cdot 1) = \sin(\pi) = 0$

no surprise here

(b) $\lim_{x\to 1} \frac{e^x(x-1)}{x^2+x-2}$.

if $x \neq 1$

$$\frac{e^x(x-1)}{x^2+x-2} = \frac{e^x(x-1)}{(x-1)(x+2)} = \frac{e^x}{x+2} \xrightarrow[x\to 1]{} \frac{e}{3}$$

(c) $\lim_{x\to 0} \frac{\sqrt{1+2x}-\sqrt{1+x}}{3x}$

(4) Either evaluate the limit or explain why it does not exist. Sketching a graph might be helpful.

(a) $\lim_{x\to 1} f(x)$ where $f(x) = \begin{cases} \sqrt{x} & 0 \le x < 1 \\ 3 & x = 1 \\ 2 - x^2 & x > 1 \end{cases}$.

(b) $\lim_{x\to 1} f(x)$ where $f(x) = \begin{cases} \sqrt{x} & 0 \le x < 1 \\ 1 & x = 1 \\ 4 - x^2 & x > 1 \end{cases}$.