# Math 100 – WORKSHEET 8
# INVERSE FUNCTIONS

### 1. More on the chain rule

(1) Suppose $f, g$ are differentiable functions with $f(g(x)) = x^3$. Suppose that $f'(g(4)) = 5$. Find $g'(4)$.

### 2. Inverse functions

To find the inverse for $y = f(x)$: (1) "solve for $x$", get $x = g(y)$ (2) "exchange $x, y$" to get $g(x)$.

(1) Find the function inverse to $y = x^7 + 3$.

(2) Does $y = x^2$ have an inverse?

(3) Consider the function $y = \sqrt{x - 1}$ on $x \geq 1$.
   (a) Find the inverse function, in the form $x = g(y)$.

   (b) Find $\frac{dy}{dx}$, $\frac{dx}{dy}$ and calculate their product.

To find the derivative of $f^{-1}$: (1) Convert $y = f^{-1}(x)$ to the form $x = f(y)$ (2) compute $\frac{dx}{dy}$ (3) In $\frac{dy}{dx}$ plug in $y = f^{-1}(x)$ to get expression in terms of $x$.

(1) Given that $\frac{d}{dy}y^2 = 2y$, find $\frac{d}{dx}\sqrt{x}$.

(2) Find $\frac{d}{dx}\arcsin x$.

(3) Find $\frac{d}{dx}\log x$.

(4) (Derivatives and logarithms)
  (a) Differentiate $\log \sqrt[k]{t}$.

  (b) (Final, 2012) Let $y = \log(\sin(\log x))$. Find $\frac{dy}{dx}$.