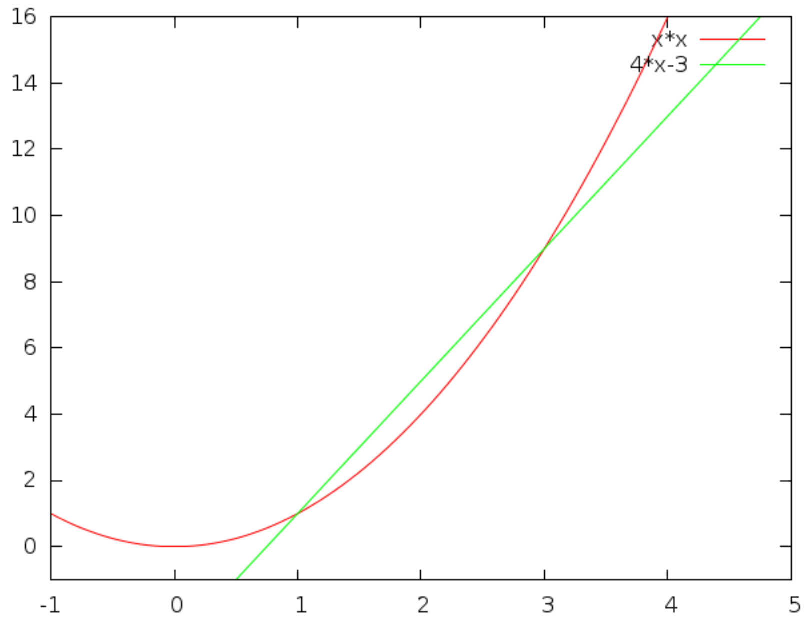
Math 100 – SOLUTIONS TO WORKSHEET 1
TANGENT AND VELOCITY PROBLEMS; LIMITS

1. The slope of a graph



(1) Find the slope of the line through $P(1,1)$ and $Q(x,x^2)$ where:
 (a) $x = 3$
 Solution: $Q = (3,9)$ so slope is $\frac{\Delta y}{\Delta x} = \frac{9-1}{3-1} = 4$
 (b) $x = 1.1$
 Solution: $Q = (1.1, 1.21)$ so slope is $\frac{\Delta y}{\Delta x} = \frac{1.21-1}{1.1-1} = \frac{0.21}{0.1} = 2.1$
 (c) $x = 1.01$
 Solution: $Q = (1.01, 1.0201)$ so slope is $\frac{\Delta y}{\Delta x} = \frac{1.0201-1}{1.01-1} = \frac{0.0201}{0.01} = 2.01$
 (d) $x = 1.001$
 Solution: $Q = (1.001, 1.002001)$ so slope is $\frac{\Delta y}{\Delta x} = \frac{1.002001-1}{1.001-1} = \frac{0.002001}{0.001} = 2.001$
 What is the slope of the tangent line at $P(1,1)$? What is its equation?
 Solution: The slope is 2, so the line is $y - 1 = 2(x-1)$ or $y = 2x - 1$.

2. Limits

(1) Evaluate $f(x) = \frac{x-3}{x^2-x-6}$ at $x = 2.9$, 2.99, 2.999, 3.1,3.01, 3.001. What is $\lim_{x\to 3} f(x)$?

 Solution: For $x \neq 3$ we have $\frac{x-3}{x^2-x-6} = \frac{x-3}{(x-3)(x+2)} = \frac{1}{x+2}$ so $\lim_{x\to 3} f(x) = \lim_{x\to 3} \frac{1}{x+2} = \boxed{\frac{1}{5}}$

(2) Evaluate
 (a) $\lim_{x\to 1} \sin(\pi x)$
 Solution: The function is nice and $\lim_{x\to 1} \sin(\pi x) = \sin(\pi) = 0$.

Date: 5/9/2019, Worksheet by Lior Silberman. This instructional material is excluded from the terms of UBC Policy 81.

(b) $\lim_{x\to 1} \frac{e^x(x-1)}{x^2+x-2}$.

Solution: $\frac{e^x(x-1)}{x^2+x-2} = \frac{e^x(x-1)}{(x-1)(x+2)} = \frac{e^x}{x+2} \xrightarrow[x\to 1]{} \frac{e^1}{1+2} = \boxed{\frac{e}{3}}$.

(c) $\lim_{x\to 0} \frac{\sqrt{1+2x}-\sqrt{1+x}}{3x}$

Solution: We have

$$\begin{aligned}
\frac{\sqrt{1+2x}-\sqrt{1+x}}{3x} &= \frac{\sqrt{1+2x}-\sqrt{1+x}}{3x} \cdot \frac{\sqrt{1+2x}+\sqrt{1+x}}{\sqrt{1+2x}+\sqrt{1+x}} \\
&= \frac{\left(\sqrt{1+2x}-\sqrt{1+x}\right)\left(\sqrt{1+2x}+\sqrt{1+x}\right)}{3x\left(\sqrt{1+2x}+\sqrt{1+x}\right)} \\
&= \frac{(1+2x)-(1+x)}{3x\left(\sqrt{1+2x}+\sqrt{1+x}\right)} \\
&= \frac{x}{3x} \cdot \frac{1}{\left(\sqrt{1+2x}+\sqrt{1+x}\right)} \\
&= \frac{1}{3\left(\sqrt{1+2x}+\sqrt{1+x}\right)} \xrightarrow[x\to 0]{} \frac{1}{3\left(\sqrt{1}+\sqrt{1}\right)} = \boxed{\frac{1}{6}}.
\end{aligned}$$

(3) Either evaluate the limit or explain why it does not exist. Sketching a graph might be helpful.

(a) $\lim_{x\to 1} f(x)$ where $f(x) = \begin{cases} \sqrt{x} & 0 \le x < 1 \\ 1 & x = 1 \\ 2-x^2 & x > 1 \end{cases}$.

Solution: From the left $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \sqrt{x} = \sqrt{1} = 1$. From the right $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 2-x^2 = 2-1 = 1$ so the limit exists and equals 1.

(b) $\lim_{x\to 1} f(x)$ where $f(x) = \begin{cases} \sqrt{x} & 0 \le x < 1 \\ 1 & x = 1 \\ 4-x^2 & x > 1 \end{cases}$.

Solution: From the left $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \sqrt{x} = \sqrt{1} = 1$. From the right $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 4-x^2 = 4-1 = 3$ so the does not exist.