Math 101 – WORKSHEET 32
MANIPULATING POWER SERIES

1. Manipulating power series: Calculus

(1) Let $f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$, $g(x) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} x^n$. We know that f converges everywhere, while g converges in $(-1, 1]$.

(a) Find the power series representation of $f'(x)$. What is $f(x)$?

(b) Find the power series representation of $g'(x)$. What is $g'(x)$? What is $g(x)$?

(c) Conclude that $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} = \log 2$.

(2) Consider the *error function* $\mathrm{erf}(x) = \int_0^x \exp(-t^2)\,\mathrm{d}t$.

(a) Find the power series expansion of $\mathrm{erf}(x)$ about zero.

(b) How many terms in the expansion are necessary to estimate $\mathrm{erf}(\frac{1}{2})$ to within 0.001?

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2. Manipulating power series: summing series

(3) Find $\sum_{n=1}^{\infty} \frac{1}{n2^n}$.

(4) Avatars of geometric series.

(a) Evaluate $\sum_{n=1}^{\infty} \frac{n}{2^n}$.

(b) Express $\sum_{n=1}^{\infty} n^2 x^n$ as a *rational function* (ratio of polynomials).

(5) Find a simple formula for $\sum_{n=0}^{\infty} \frac{e^{nx}}{n!}$.