# Math 101 – WORKSHEET 23
# SERIES

## 1. Tool: Squeeze Theorem

(1) Determine if each sequence is convergent or divergent. If convergent, evaluate the limit.
   (a) (Final 2013) $\left\{(-1)^n \sin\left(\frac{1}{n}\right)\right\}_{n=1}^{\infty}$.

   (b) (Final 2011) $\left\{\frac{\sin(n)}{\log(n)}\right\}_{n=2}^{\infty}$ (why do we have $n \geq 2$ here?)

   (c) (Math 105 Final 2012) $a_n = 1 + \frac{n! \sin(n^3)}{(n+1)!}$.

## 2. Skill 1: Geometric series and decimal expansions

(1) (Final 2013) Find the sum of the series $\sum_{n=2}^{\infty} \frac{3 \cdot 4^{n+1}}{8 \cdot 5^n}$. Simplify your answer.

(2) Express each decimal expansion using a geometric series, sum the series, then simplify to obtain a rational number.
   (a) $0.333333\ldots$

   (b) $0.5757575757\ldots$

   (c) $0.6545454545454\ldots$

*Date*: 6/3/2017, Worksheet by Lior Silberman. This instructional material is excluded from the terms of UBC Policy 81.

## 3. Skill 2: Telescoping series

(3) Write an expression for the partial sums, decide if the series converges, and if so determine the sum.

(a) (Final 2015) $\sum_{n=3}^{\infty} \left( \cos\left(\frac{\pi}{n}\right) - \cos\left(\frac{\pi}{n+1}\right) \right)$

(b) $\sum_{n=1}^{\infty} \left( n^2 - (n+1)^2 \right)$

(c) $\sum_{n=1}^{\infty} \frac{2}{n(n+2)}$

(d) $\sum_{n=0}^{\infty} \left( \arctan(n) - \arctan(n+1) \right)$