

**Math 101 – WORKSHEET 22**  
**SEQUENCES**

1. SKILL 1: EXPRESSIONS FOR SEQUENCES

(1) For each of the following sequences, write a formula for the general term

(a)  $\{1, 2, 3, 4, 5, 6, \dots\}$

(b)  $\{1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \frac{1}{36}, \dots\}$

(c)  $\{3, 7, 11, 15, 19, \dots\}$

(d)  $\{\frac{7}{9}, \frac{7}{27}, \frac{7}{81}, \frac{7}{243}, \frac{7}{729}, \frac{7}{3187}, \dots\}$

(e)  $\{\frac{1}{2}, \frac{1}{2}, \frac{3}{8}, \frac{1}{4}, \frac{5}{32}, \frac{3}{32}, \frac{7}{128}, \frac{1}{32}, \frac{9}{512}, \frac{5}{512}, \dots\} = \{\frac{1}{2}, \frac{2}{4}, \frac{3}{8}, \frac{4}{16}, \frac{5}{32}, \frac{6}{64}, \frac{7}{128}, \frac{8}{256}, \frac{9}{512}, \frac{10}{1024}, \dots\}$

(f)  $\{1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, \dots\}$

(g)  $\{0, \frac{3}{8}, \frac{2}{27}, \frac{5}{64}, \frac{4}{125}, \frac{7}{216}, \frac{6}{343}, \frac{9}{512}, \frac{8}{729}, \frac{11}{1000}, \dots\}$

2. SKILL 2: LIMITS OF SEQUENCES

(2) Determine if the sequences is convergent or divergent. If convergent, evaluate the limit.

(a)  $\{\frac{1}{n}\}_{n=1}^{\infty}$

(b)  $\{\frac{n}{n+1}\}_{n=1}^{\infty}$

(c)  $\{\sin(n)\}_{n=5}^{\infty}$

(d)  $\{\sin(\frac{1}{n})\}_{n=1}^{\infty}$

(3) Further problems

(a) Does  $\lim_{n \rightarrow \infty} \frac{n}{\sqrt{n+1000}}$  exist?

(b)  $\lim_{n \rightarrow \infty} \frac{n}{2^n} =$

(c) (Math 103 final, 2014) Consider the sequence  $\{a_n\}_{n=1}^{\infty} = \{1, 0, \frac{1}{2}, 0, 0, \frac{1}{3}, 0, 0, 0, \frac{1}{4}, 0, 0, 0, 0, \frac{1}{5}, \dots\}$ .  
Decide whether  $\lim_{n \rightarrow \infty} a_n = 0$ .

### 3. TOOL: SQUEEZE THEOREM

(4) Determine if each sequence is convergent or divergent. If convergent, evaluate the limit.

(a) (Final 2013)  $\{(-1)^n \sin(\frac{1}{n})\}_{n=1}^{\infty}$ .

(b) (Final 2011)  $\left\{\frac{\sin(n)}{\log(n)}\right\}_{n=2}^{\infty}$  (why do we have  $n \geq 2$  here?)

(c) (Math 105 Final 2012)  $a_n = 1 + \frac{n! \sin(n^3)}{(n+1)!}$ .