MATH 100 – WORKSHEET 13
RELATED RATES AND THE LINEAR APPROXIMATION

1. Related rates

(1) A particle is moving along the curve $y^2 = x^3 + 2x$. When it passes the point $\left(1, \sqrt{3}\right)$ we have $\frac{\mathrm{d}y}{\mathrm{d}t} = 1$. Find $\frac{\mathrm{d}x}{\mathrm{d}t}$.

(2) Two ships are travelling near an island. The first is located 20km due west of it and is moving due north at 5km/h. The second is located 15km due south of it and is moving due south at 7km/h. How fast is the distance between the ships changing?

(3) The same setting, but now the first ship is moving toward the island.

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(4) A conical drain is 6m tall and has radius 1m at the top.
   (a) The drain is clogged, and is filling up with rain water at the rate of $5\text{m}^3/\text{min}$. How fast is the water rising when its height is 5m?
   (b) The drain is unclogged and water begins to clear at the rate of $15\text{m}^3/\text{min}$ (but rain is still falling). At what height is the water falling at the rate of 40m/min?

2. The Linear Approximation

Fact. *For x near a we have $f(x) \approx L(x)$ where*

$$\boxed{L(x) = f(a) + f'(a)(x - a)}$$

(1) Use a linear approximation to estimate
   (a) $\sqrt{1.2}$

   (b) $(15)^{1/4}$

   (c) $\log 3$