Math 322: Group Theory
Fall Term, 2014

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Course Website	`http://www.math.ubc.ca/~lior/teaching/1415/322_F14/`
Contact me at	MAT 229B — 604-827-3031 – `lior@math.ubc.ca`
My Website	`http://www.math.ubc.ca/~lior/`
Class	TTh 14:00-15:30, LSK 460
Office Hours	TTh 9:30-10:30, W 10:30-11:30
Textbook	None required; see below.
Course Prerequisites	(a) Score$\geq$ 68% in one of MATH 223,310 or (b) one of MATH 152,221,223 and score$\geq$ 80% in MATH 223

About the course

Summary and textbook

This is a first course in the abstract algebra sequence, intended for honours students. We will learn the basics of the theory of groups. The sequence is continued by Math 323 (ring theory) in the spring, and by Math 422 (field and Galois theory) and 423 (commutative and homological algebra) the following year. Students who wish to acquire a textbook should choose from the following three options:

1. *Abstract Algebra* by Dummit and Foote is the recommended textbook for both 322 and 323 this year (it contains the material for Math 322, 323, 422 and most of 423, in fact). Buy it if you want a single abstract algebra book.
2. *Contemprorary Abstract Algebra* by Gallian is a gentler alternative to Dummit and Foote. It is less dense and gives more details.
3. *An Introduction to the Theory of Groups* by Rotman (GTM 148) covers group theory in much greater depth. You can download a PDF for free from the publisher (follow the link from the course website; you must be logged on to the UBC network for this), or of course buy a hardcopy. In the instructor's opinion Rotman's book is the best introductory textbook on group theory.

The material of this course is actually standard, and any book titled "group theory" or "abstract algebra" will include everything.

Why Groups?

Groups are the mathematical objects which encode *symmetry*, and a standard practice in mathematics is learning about an object from the algebraic behaviour of its symmetry group. An example of this pattern is the use of change-of-basis matrices in linear algebra (the object being studied is the vector space, and the invertible matrix is a symmetry). Deeper

examples come from physics, where symmetries of the world such as translation (shifting the experimental apparatus), rotation (turning it around) and time-translation (holding the experiment at a different time) are intimately connected to conservation laws (conservation of momentum, angular momentum, and energy, respectively). The atomic structure of crystals is understood in terms of the *crystallographic groups* (describing the symmetries of the arrangements of atoms in a solid). Some of the strangest aspects of quantum mechanics are connected to what happens when the positions of identical particles are swapped (surprisingly, nature can sometimes tell!). Groups are also useful computational tools (most modern encryption schemes rely on computation in a group, for example).

Groups are also the simplest algebraic objects. Understanding them is a pre-requisite for learning more algebra, or any mathematics that relies on algebra. Along the way studying them will give some new perspectives on the linear algebra students have already learned.

Rough course plan

1. Introductory examples.
2. Groups and homomorphism.
3. Quotients and the isomorphism theorems.
4. Group actions.
5. The Sylow Theorems:.
6. Other topics as time permits.

Metamathematical goals

For most students this will be their first or second course in formal, abstract mathematics. As such, there are several important learning outcomes beyond simply "learning the material". These include:

- Students will be able to tell if written arguments are correct or not, and construct counterexamples.
- Students will be able to express their ideas coherently and elegantly.
- Students will occasionally solve problems requiring new ideas.

Teaching and learning

Significant prerequisites

- Basic linear algebra: vector spaces, subspaces and linear maps. Invertibility of matrices.
- Proofs (at the level of 223 or 220): constructing simple proofs and checking them for correctness.

What you can expect from me

- Demanding homework and examinations.
  - It is impossible to learn rigorous mathematics without practising it. The transition to rigorously solving homework problems is not easy, but is necessary for students to progress in mathematics.
  - All students will be progressing together; I will post grade statistics for all to see.

  - Homework (and possible exam) marks will be scaled at the end of the course to account for the difficulty.

- Responses to your questions and concerns: continuously in class and during my office hours, within reasonable time by e-mail outside class.
- Clear explanations of what is correct in your work and what is not, and how you can improve.

What's expected from you

- Come prepared to class. You will need to read relevant material in a textbook – information will be posted on the course website.
- Actively participate in the course: do some reading, think about the material, solve the problem sets, and ask questions.
  - Asking questions when you don't understand, or want to learn more, ensures that you get what you want out of the course. Ask me questions in class, by email, and during office hours. Also, ask your colleagues questions outside of class – you will both benefit from the discussion!
  - Working on the problem sets is *absolutely essential* for learning the material. **It is extremely rare for students who skip problem sets to do well on exams.**
  - Practice problems in the homework are also important – they are important for getting a "concrete feel" for the abstract mathematics we will be doing.
  - I may call on you in class.
- Submit written work that is readable and communicates your ideas.
  - Reasoning needs to be conveyed in English sentences, not as a sequence of formulas.
  - Unnecessarily convoluted writing may be marked down.
  - If you need help with this beyond my office hours, try the Math Learning Centre.

Official Policies

General policies

- **Late or missed exams and assignments will not be accepted for credit and will be given a grade of zero**. In exceptional circumstances (a proof of the emergency is required, and advance notification if possible will be required) the missed work will be registered (and not count toward the average of that part of the course) if you finish it and hand it in after the emergency has passed.
- All assertions require ***proof*** unless the problem explicitly states otherwise. No matter the operative word ("find", "solve", "establish", "calculate", "determine" ...), you must justify your answer.
- Written work should be presented carefully, in complete English sentences, and with sufficient detail. A "correct sequence of formulas" will only merit partial credit. Examples of the expectations may be distributed together with the first problem set.
- The instructor may designate material (e.g. definitions) for self-study, in which case you are responsible for learning this material before it is used in class and in problem sets.

Homework

- There will be up to twelve problem sets posted to the course website, due at the *beginning* of class on the day shown. The lowest two scores will be dropped when calculating the homework grade.
  - Problems will focus on conceptual material, with some calculational problems.
  - You are encouraged to work on solving the problems together. However, each of you must write your solutions independently, in your own words. You may (and should) share your ideas but *you may not share your written work.*
  - It is possible that only certain problems from a problem set will be selected for grading.
  - To the extent possible solutions will be posted on the secure (Connect) website.

Exams

- There will one midterm exam in class; its date will be announced later.
  - If you need special accommodations when taking written exams, please contact the Office of Access & Diversity (`access.diversity@ubc.ca`).
  - If the midterm (or final) exam conflicts with a religious observance, or if you have any other legitimate conflict, please contact me *at least two weeks ahead of time* so we can make appropriate arrangements.
- There will be a final exam during the usual exam period.

Final grade

- The final grade will be calculated as follows:

Problem sets: 30%
Midterm: 20%
Final exam: 50%

References

[1] Dummit and Foote, *Abstract Algebra*

[2] Gallian, *Contemporary Abstract Algebra*

[3] Rotman, *Introduction to Theory of Groups*