MATH 253 – WORKSHEET 20
ITERATED INTEGRALS ON RECTANGLES

Theorem (Fubini). *Let $f(x,y)$ be integrable on the rectangle $R = [a,b] \times [c,d]$. Then*

$$\iint\limits_{R} f(x,y)\,\mathrm{d}x\,\mathrm{d}y = \int_{y=c}^{y=d} \mathrm{d}y \left(\int_{x=a}^{x=b} \mathrm{d}x\, f(x,y) \right) = \int_{x=a}^{x=b} \mathrm{d}x \left(\int_{y=c}^{y=d} \mathrm{d}y\, f(x,y) \right)$$

(1) Integrate $f(x,y) = (1-y)x$ on $[2,3] \times [4,5]$.

(2) Integrate $f(x,y) = x(y + x^2)$ on $[0,1] \times [0,1]$.

(3) Evaluate $\iint_{[-1,1]\times[0,1]} \frac{y\sin x}{1+\cos^2 y}\, dx\, dy$. What is the integral of $f(x,y) = (x+y)\, e^{-x^4-y^4}$ on the plane?

(4) Find the average value of $f(x,y) = e^y\sqrt{x+e^y}$ over the rectangle $[0,4]\times[0,1]$.