MATH 253 – WORKSHEET 19
INTEGRATION ON RECTANGLES

Let $f(x,y)$ be defined on a region R. Approximately divide the region R into small rectangles around sample points (x_i, y_j) of size Δx_i by Δy_j. Then

$$\iint_R f(x,y)\,\mathrm{d}x\,\mathrm{d}y = \lim_{N,M\to\infty} \sum_{i=1}^{N}\sum_{j=1}^{M} f(x_i,y_j)\Delta x_i \Delta y_j$$

$\Delta x_i \Delta y_j$ is exactly the area of the small rectangle, so $f(x_i,y_i)\Delta x_i \Delta y_j$ is approximately the volume of the part of the solid above this small rectangle.

Example 1. Let A be the solid lying above the rectangle $R = [0,2]\times[0,3]$ and below the graph of $z = x+y$. Approximate the volume of A by:

(1) Dividing R into 4 equal rectangles and using the midpoints.

(2) Dividing R into 6 equal squares and using the lower left corners.

(3) Dividing R into 6 equal squares and using the midpoints.

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