MATH 253 – WORKSHEET 19
INTEGRATION ON RECTANGLES

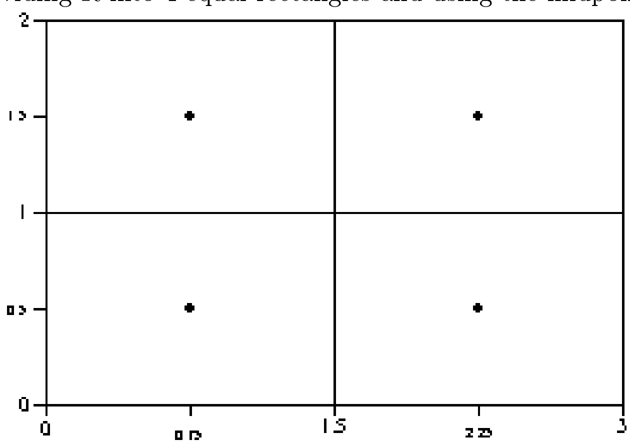
Let $f(x,y)$ be defined on a region R. Approximately divide the region R into small rectangles around sample points (x_i, y_j) of size Δx_i by Δy_j. Then

$$\iint_R f(x,y)\,\mathrm{d}x\,\mathrm{d}y = \lim_{N,M\to\infty} \sum_{i=1}^{N}\sum_{j=1}^{M} f(x_i,y_j)\Delta x_i \Delta y_j$$

$\Delta x_i \Delta y_j$ is exactly the area of the small rectangle, so $f(x_i, y_i)\Delta x_i \Delta y_j$ is approximately the volume of the part of the solid above this small rectangle.

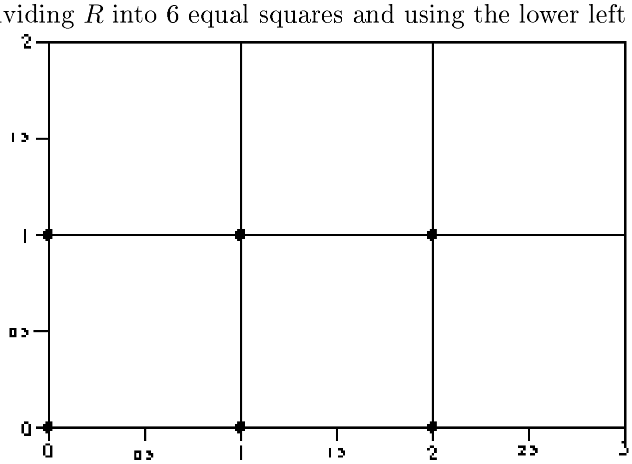
Example 1. Let A be the solid lying above the rectangle $R = [0,3]\times[0,2]$ and below the graph of $z = x+y$. Approximate the volume of A by:

(1) Dividing R into 4 equal rectangles and using the midpoints.



each little rectangle has area $\frac{3}{2}\times 1 = 1$, so volume
$\approx (0.75+0.5)\,\frac{3}{2} + (2.25+0.5)\,\frac{3}{2} + (0.75+1.5)\,\frac{3}{2} + (2.25+1.5)\,\frac{3}{2} = 15.$

(2) Dividing R into 6 equal squares and using the lower left corners.



each little rectangle has area $1\times 1 = 1$, so volume
$\approx (0+0) + (1+0) + (2+0) + (0+1) + (1+1) + (2+1) = 11.$

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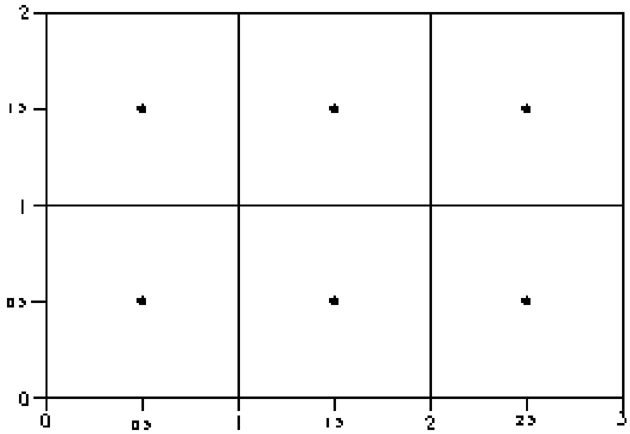
(3) Dividing R into 6 equal squares and using the midpoints.



each little rectangle has area $1 \times 1 = 1$, so volume
$\approx (0.5 + 0.5) + (1.5 + 0.5) + (2.5 + 0.5) + (0.5 + 1.5) + (1.5 + 1.5) + (2.5 + 1.5) = 15.$

Remark. The exact volume happens to be 15.