MATH 253 – WORKSHEET 17
LAGRANGE MULTIPLIERS

1. Optimization

1.1. **Ordinary optimization.** Suppose we want to find the maximum or minimum of $f(x, y)$ in a region R. We solve the system of equations $\vec{\nabla} f(x_0, y_0) = \vec{0}$ to find the *critical points*, and then evaluate f at critical points and on the boundary of R.

1.2. **Constrained optimization.** Suppose we want to find the maximum or minimum of $f(x, y)$ *subject to the constraint* $g(x, y) = 0$. **Fact**: any local maximum/minimum *on the level set of* g occurs at a point (x_0, y_0) where $\vec{\nabla} f$ is proportional to $\vec{\nabla} g$. In other words, to find local maxima/minima we solve the system of equations

$$\begin{cases} \frac{\partial f}{\partial x}(x_0, y_0) & = \lambda \frac{\partial g}{\partial x}(x_0, y_0) \\ \frac{\partial f}{\partial y}(x_0, y_0) & = \lambda \frac{\partial g}{\partial y}(x_0, y_0) \\ g(x_0, y_0) & = 0 \end{cases}$$

where the unknowns are x_0, y_0, λ.

2. Problems

(1) Find the equation of the plane which passes through $(1, 2, 3)$ and encloses the smallest volume in the positive octant.

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(2) Find the absolute max and min of $f(x, y) = x^3y^2 - 2y^4x + 2x$ on $\{x^2 + y^2 \leq 4\}$.