

# Math 613D: Modular Forms

## Fall Term, 2010

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v0.9 (August 30, 2010)

Main Course Website	<a href="http://www.math.ubc.ca/~lior/teaching/1011/613D_F10/">http://www.math.ubc.ca/~lior/teaching/1011/613D_F10/</a>
SLATE Website	TBD
Contact me at	MAT 229B — 604-827-3031 — <a href="mailto:lior@math.ubc.ca">lior@math.ubc.ca</a>
My Website	<a href="http://www.math.ubc.ca/~lior/">http://www.math.ubc.ca/~lior/</a>
Class	M 10-12, W 11-12 at MATH 202
Office Hours	TBD
Textbook (optional)	Miyake (ISBN: 978-3540295921)
Course Prerequisites	See below

## About the course

We will define holomorphic modular forms and study their analytic properties.

I will generally follow the book *Modular Forms* by Miyake [4]. You can buy yourself a copy, or download the PDF from SpringerLink at <http://springerlink.com/content/978-3-540-29592-1/contents/>. However, the material is standard and covered in many books – feel free to use any reference that suits you; examples include [1, 2, 5]. A standard reference for hyperbolic geometry is [3].

I will assign problem sets during the course, mainly to review material I will use in lectures or to develop ideas that have been left out of the lectures.

## Official Policies

### Pre-requisites

You'll need to know complex analysis, basic number theory (especially Dirichlet characters and the theory of the Riemann zetafunction) and some group theory. You will get more out of the course if you happen to be familiar with any of algebraic topology, representation theory, the theory of Riemann surfaces, or algebraic geometry.

### Assessment

The grade will be based on the regular problem sets; they will normally be due one or two weeks after they were assigned at the start of the Wednesday morning lecture.

- Late assignments will not be accepted for credit. In exceptional circumstances (a proof of the emergency and advance notification if possible will be required) a late problem set will be registered (that is, will not be scored a zero) if you finish it and hand it in after the emergency has passed.

- You are encouraged to work on solving the problems together. However, each of you must write your solutions independently. You may (and should) share your ideas but you may not share your written work.
- Problem sets will be available from the course website; solutions will be posted on a companion SLATE site.

## Tentative outline

1. Introduction: doubly period functions, the space of lattices and Eisenstein series. Sums of squares and theta series.
2. Hyperbolic geometry and Fuchsian groups.
3. Modular forms.
4. Hecke operators.
5. Hecke L-functions and Weil's converse theorem.
6. More advanced results as time allows, depending on audience interests. Possibilities include:
  - Bounds on Fourier coefficients.
  - Bounds on L-functions (introduction to subconvexity).
  - The Rankin-Selberg L-function.
  - Non-holomorphic modular forms
  - The representation-theoretic point of view.
  - Arithmetic theory of modular forms; the modularity theorem.

## References

- [1] Fred Diamond and Jerry Shurman. *A first course in modular forms*, volume 228 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 2005.
- [2] R. C. Gunning. *Lectures on modular forms*. Notes by Armand Brumer. Annals of Mathematics Studies, No. 48. Princeton University Press, Princeton, N.J., 1962.
- [3] Svetlana Katok. *Fuchsian groups*. Chicago Lectures in Mathematics. University of Chicago Press, Chicago, IL, 1992.
- [4] Toshitsune Miyake. *Modular forms*. Springer Monographs in Mathematics. Springer-Verlag, Berlin, english edition, 2006. Translated from the 1976 Japanese original by Yoshitaka Maeda.
- [5] Goro Shimura. *Introduction to the arithmetic theory of automorphic functions*, volume 11 of *Publications of the Mathematical Society of Japan*. Princeton University Press, Princeton, NJ, 1994. Reprint of the 1971 original, Kanô Memorial Lectures, 1.