Math 100:V02 – WORKSHEET 13
QUALITATIVE ASPECTS OF DIFFERENTIAL EQUATIONS

1. Fixed points

(1) (Review)
   (a) For which value of ω is $y = A\sin(\omega t) + B\cos(\omega t)$ a solution of $\ddot{y} = -9y$?

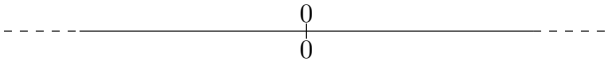
   (b) Can you find the general solution of $\ddot{y} = 9y$?

(2) (Steady states = fixed points = equilibria)
   (a) Consider the Malthusian growth equation $\dot{y} = ry$, $r > 0$. Can you find a value a so that $y(t) \equiv a$ is a solution?

   (b) What about the *logistic growth* model $\dot{y} = ry(1-y)$ *with* $r > 0$*?*

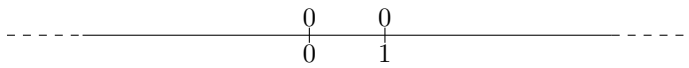
   (c) What about $\dot{y} = y^3 - 5y^2 + 6y$?

(3) (Phase line)
   (a) In the model $\dot{y} = ry$ with $r > 0$, what is the sign of $\dot{y}$ when $y < 0$? when $y > 0$? What would the solution look like if we started with y_0 in each range? Draw the phase line.

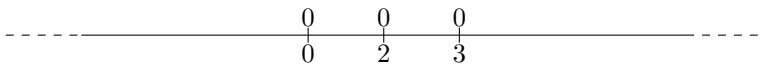


Date: 5/3/2024, Worksheet by Lior Silberman. This instructional material is excluded from the terms of UBC Policy 81.

(b) What about the *logistic growth* model $\dot{y} = ry(1-y)$*?*



(c) What about $\dot{y} = y^3 - 5y^2 + 6y$?



(4) Analyze $\frac{dy}{dt} = -2y^3 + 9y^2 - 12y$

2. Taylor expansion

(5) Consider the equation $\dot{y} = -\sin y$, $y(0) = \frac{\pi}{2}$.

(a) What is $\dot{y}(0)$?

(b) What is $\ddot{y}(0)$?

(c) What is the third-order Taylor expansion of y about $t = 0$?

(d) What are the fixed points of this equation? Are the stable or unstable?