Math 100:V02 – WORKSHEET 11
INVERSE TRIG; LOGARITHMIC DIFFERENTIATION

1. Logarithmic differentiation

Fact. $(f(g(x)))' = f'(g(x))\,g'(x)$ *or* $\frac{\mathrm{d}}{\mathrm{d}x}(f(g(x))) = \frac{df}{dg}\cdot\frac{dg}{dx}$; *also* $\frac{d}{dx}\log x = \frac{1}{x}$.

$\log_b(b^x) = b^{\log_b x} = x$ $\qquad$ $\log_b(xy) = \log_b x + \log_b y$ $\qquad$ $\log_b(x^y) = y\log_b x$ $\qquad$ $\log_b \frac{1}{x} = -\log_b x$

(1) Differentiate

(a) $\frac{\mathrm{d}(\log(ax))}{\mathrm{d}x} =$ $\qquad$ $\frac{\mathrm{d}}{\mathrm{d}t}\log\left(t^2+3t\right) =$

(b) $\frac{\mathrm{d}}{\mathrm{d}x}x^2\log(1+x^2) =$ $\qquad$ $\frac{\mathrm{d}}{\mathrm{d}r}\frac{1}{\log(2+\sin r)} =$

(2) (Logarithmic differentiation) Use $\log(fg) = \log f + \log g$ to differentiate $y = \left(x^2+1\right)\cdot\sin x\cdot\frac{1}{\sqrt{x^3+3}}\cdot e^{\cos x}$.

(3) Differentiate using $f' = f \times (\log f)'$

(a) x^n

(b) x^x

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(c) $(\log x)^{\cos x}$

(d) (Final, 2014) Let $y = x^{\log x}$. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x only.

(4) Let $f(x) = g(x)^{h(x)}$. Find a formula for f' in terms of g' and h'.

2. Inverse trig

(5) (evaluation)

(a) (Final 2014) Evaluate $\arcsin\left(-\frac{1}{2}\right)$; Find $\arcsin\left(\sin\left(\frac{31\pi}{11}\right)\right)$.

(b) (Final 2015) Simplify $\sin(\arctan 4)$

(c) Find $\tan\left(\arccos\left(0.4\right)\right)$

(6) Let $f(\theta) = \sin^2\theta + \cos^2\theta$. Find $\frac{df}{d\theta}$ without using trigonometric identities. Evaluate $f(0)$ and conclude that $\sin^2\theta + \cos^2\theta = 1$ for all θ.

(7) (Inverse functions)

(a) Suppose $g(x) = e^x$, $f(y) = \log y$. Show that $f(g(x)) = x$ and conclude that $(\log y)' = \frac{1}{y}$.

(b) Let $\theta = \arcsin x$. Find $\frac{d\theta}{dx}$. *Hint*: solve for x first.

(8) Differentiation

(a) Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(\arcsin\left(2x\right)\right)$

(b) Find the line tangent to $y = \sqrt{1 + (\arctan(x))^2}$ at the point where $x = 1$.

(c) Find y' if $y = \arcsin\left(e^{5x}\right)$. What is the domain of the functions y, y'?