Math 100A – WORKSHEET 6
APPLICATIONS OF THE CHAIN RULE

1. Review

(1) Differentiate
   (a) ⋆ $e^{\sqrt{\cos x}}$

(2) (Final, 2014) ⋆ Let $y = x^{\log x}$. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x only.

2. Implicit Differentiation

(3) Find the line tangent to the curve $y^2 = 4x^3 + 2x$ at the point $(2, 6)$.

(4) (Final, 2015) Let $xy^2 + x^2y = 2$. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point $(1, 1)$.

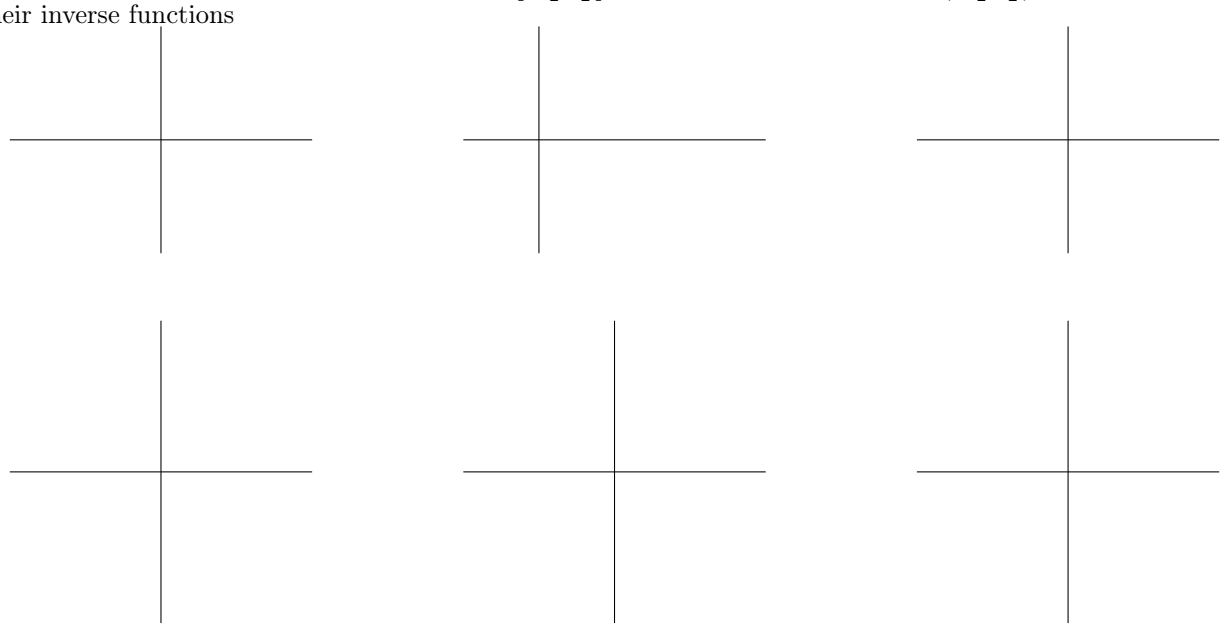
(5) (Final 2012) Find the slope of the line tangent to the curve $y + x\cos y = \cos x$ at the point $(0, 1)$.

(6) Find y'' (in terms of x, y) along the curve $x^5 + y^5 = 10$ (ignore points where $y = 0$).

Date: 11/10/2023, Worksheet by Lior Silberman. This instructional material is excluded from the terms of UBC Policy 81.

3. Inverse trig functions

(7) Draw on the following axes graphs of $\sin\theta$ on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $\cos\theta$ on $[0, \pi]$ and $\tan\theta$ on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, then of their inverse functions



(8) Evaluation

(a) (Final 2014) Evaluate $\arcsin\left(-\frac{1}{2}\right)$ and $\arcsin\left(\sin\left(\frac{31\pi}{11}\right)\right)$.

(b) (Final 2015) Simplify $\sin(\arctan 4)$

(c) Find $\tan\left(\arccos\left(0.4\right)\right)$

(9) Differentiation

(a) Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(\arctan x\right)$

(b) Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(\arcsin\left(2x\right)\right)$

(c) Find the line tangent to $y = \sqrt{1 + (\arctan(x))^2}$ at the point where $x = 1$.

(d) Find y' if $y = \arcsin\left(e^{5x}\right)$. What is the domain of the functions y, y'?

4. Related Rates

(10) A particle is moving along the curve $y^2 = x^3 + 2x$. When it passes the point $\left(1, \sqrt{3}\right)$ we have $\frac{\mathrm{d}y}{\mathrm{d}t} = 1$. Find $\frac{\mathrm{d}x}{\mathrm{d}t}$.

(11) (Final, 2015, variant) A conical tank of water is 6m tall and has radius 1m at the top.

(a) The drain is clogged, and is filling up with rainwater at the rate of $5\text{m}^3/\text{min}$. How fast is the water rising when its height is 5m?

(b) The drain is unclogged and water begins to drain at the rate of $(5 + \frac{\pi}{4})\text{m}^3/\text{min}$ (but rain is still falling). At what height is the water falling at the rate of 1m/min?