Math 100 – WORKSHEET 10
LOGARITHMIC AND IMPLICIT DIFFERENTIATION

1. Review of Logarithms

$$\boxed{\log_b(b^x) = b^{\log_b x} = x}$$

$$\boxed{\log_b(xy) = \log_b x + \log_b y} \qquad \boxed{\log_b(x^y) = y\log_b x} \qquad \boxed{\log_b \frac{1}{x} = -\log_b x}$$

(1) $\log\left(e^{10}\right) =$ $\log(2^{100}) =$ (in terms of $\log 2$)

(2) A variant on *Moore's Law* states that computing power doubles every 18 months. Suppose computers today can do N_0 operations per second.

(a) Write a formula predicting the future:

- Computers t years from now will be able to do $N(t)$ operations per second where

$$N(t) =$$

(b) A computing task would take 10 years for today's computers. Suppose we wait 3 years and then start the computation. When will we have the answer?

(c) At what time will computers be powerful enough to complete the task in 6 months?

2. Differentiation

$$\boxed{(\log x)' = \frac{1}{x}}$$

(1) Differentiate

(a) $\frac{\mathrm{d}(\log(ax))}{\mathrm{d}x} =$ $\frac{\mathrm{d}}{\mathrm{d}t}\log\left(t^2+3t\right) =$

(b) $\frac{\mathrm{d}}{\mathrm{d}x}x^2\log(1+x^2) =$ $\frac{\mathrm{d}}{\mathrm{d}r}\frac{1}{\log(2+\sin r)} =$

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(2) (Logarithmic differentiation) Use $\log(fg) = \log f + \log g$ to differentiate $y = \left(x^2+1\right) \cdot \sin x \cdot \frac{1}{\sqrt{x^3+3}} \cdot e^{\cos x}$.

(3) Differentiate using $\boxed{f' = f \times (\log f)'}$

(a) x^x

(b) $(\log x)^{\cos x}$

(c) (Final, 2014) Let $y = x^{\log x}$. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x only.

3. Implicit Differentiation

(1) Find the line tangent to the curve $y^2 = 4x^3 + 2x$ at the point $(2, 6)$.

(2) (Final, 2015) Let $xy^2 + x^2y = 2$. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point $(1, 1)$.

(3) (Final 2012) Find the slope of the line tangent to the curve $y + x\cos y = \cos x$ at the point $(0, 1)$.

(4) Find y'' (in terms of x, y) along the curve $x^5 + y^5 = 10$ (ignore points where $y = 0$).

(5) Find y' if $(x+y)\sin(xy) = x^2$.