16. Minima and Maxima (4/11/2021)

Goals.

(1) Global and local extrema
(2) Critical and singular points
(3) Finding minima and maxima using differentiation
(4) Midterm!

Last Time. Lagrange Remainder for Taylor expansion:

$$f(x) = f(a) + \underbrace{\frac{f^{(1)}(a)}{1!}(x-a)^1 + \dots + \frac{f^{(n)}(a)}{n!}(x-a)^n}_{T_n(x)} + \underbrace{\frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1}}_{R_n(x)}$$

c between a, x.

Saw how to convert estimates on $f^{(n+1)}$ to error estimates

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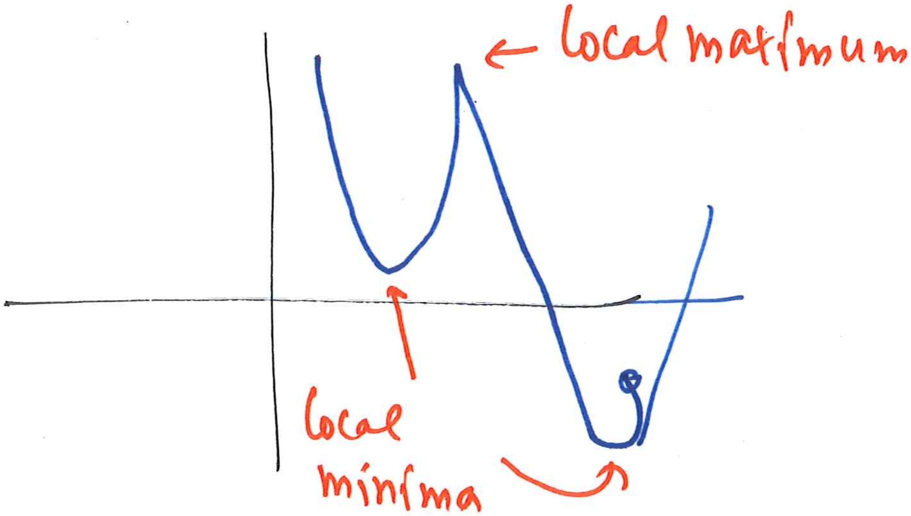
Extrema of functions



"local" max/min pt where nearby (on both sides) fcn does not exceed (go ~~above~~ below) the value at pt.

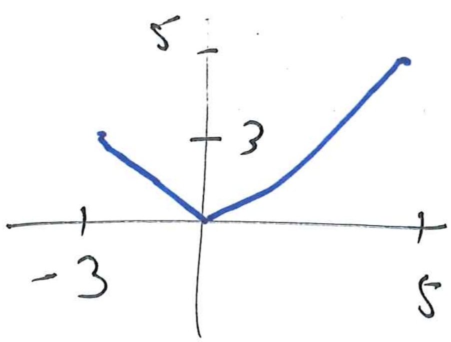
"global"/ "absolute" min/max: extreme value over domain

Math 100 – WORKSHEET 16
MINIMA AND MAXIMA

1. Absolute minima and maxima by hand

Theorem. *If f is continuous on $[a, b]$ it has an absolute maximum and minimum there.*

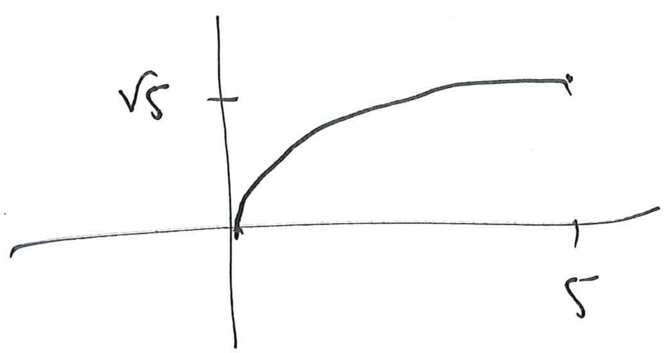
(1) Find the absolute maximum and minimum values of $f(x) = |x|$ on the interval $[-3, 5]$.



absolue max value is 5
absolue min value is 0

(2) Find the absolute maximum and minimum of $f(x) = \sqrt{x}$ on $[0, 5]$.

max is $\sqrt{5}$ achieved at $x = 5$
min is 0, " " $x = 0$



Date: 4/11/2021, Worksheet by Lior Silberman. This instructional material is excluded from the terms of UBC Policy 81.

Language: one maximum, many maxima

minimum minima

extremum extrema

Facts: (1) If f cts on $[a,b]$ then f achieves a global min & max.

(2) If f has a local extremum at x_0 and $f'(x_0)$ exists then $f'(x_0) = 0$

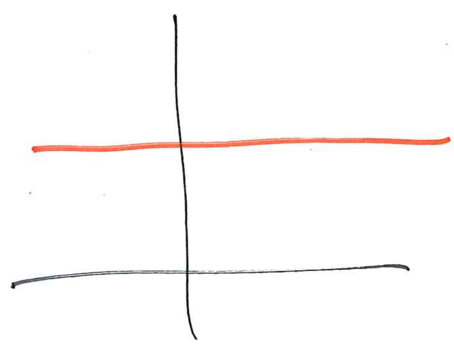
$\Rightarrow$ If f cts, extrema only occur at:

(1) critical points : $f'(x_0) = 0$

(2) singular points : $f'(x_0)$ DNE

(3) end points of domain (if any)

aside: constant fcn



2. Derivatives and local extrema

Theorem (Fermat). *If, in addition, f is defined near c (on both sides!), is differentiable at c, and has a local extremum at c then $f'(c) = 0$.*

Procedure

- Call c a *critical point* or *critical number* if $f'(c) = 0$, a *singular point/number* if $f'(c)$ does not exist.
- To find absolute maximum/minimum of a continuous function f defined on $[a, b]$:
  - Evaluate $f(c)$ at all critical and singular point.
  - Evaluate $f(a), f(b)$.
  - Choose largest, smallest value.

(3) (Final, 2011) Let $f(x) = 6x^{1/5} + x^{6/5}$.

(a) Find the critical numbers and singularities of f.

$f'(x) = 6 \cdot \frac{1}{5} \cdot x^{-4/5} + \frac{6}{5} x^{1/5} = \frac{6}{5} \frac{1+x}{x^{4/5}}$

$\Rightarrow$ singular pt at $x = 0$

$\Rightarrow$ critical pt at $x = -1$.

(b) Find its absolute maximum and minimum on the internal $[-32, 32]$.

~~Have~~ Only need to check critical, singular, and end points (because interval is closed)

$f(-1) = 6\cdot(-1)^{1/5} + ((-1)^{1/5})^6 = -6+1 = -5$

$f(0) = 6\cdot 0^{1/5} + (0^{1/5})^6 = 0$

$f(32) = 6\cdot 32^{1/5} + (32^{1/5})^6 = 6\cdot 2 + 64 = 76$

$f(-32) \qquad = -6\cdot 2 + 64 = 52$

max value: 76 (achieved at $x = 32$)

min value: -5 (" " $x = -1$)

(4) (Final, 2015) Find the critical points of $f(x) = e^{x^3 - 9x^2 + 15x - 1}$