Math 422/501: Galois Theory
Fall Semester, 2020

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Course Website	`http://www.math.ubc.ca/~lior/teaching/2021/501_F20/`
Contact me at	MATX 1112 — 604-827-3031 – lior@math.ubc.ca
My Website	`http://www.math.ubc.ca/~lior/`
Class	MW 14:00-14:50, Problem session TBA, on Zoom.
Office Hours	TBA
Textbook	None required; see below.
Course Prerequisites	A course in Group Theory and a course in Ring Theory, equivalent to UBC MATH 322 and 323.

About the course

We will study Galois Theory.

Its initial development (most significantly, by Galois) dates to the early 19th century and lies at the roots of modern abstract algebra. After a brief review of group and ring theory we will turn to the theory of fields and their extensions, especially its connection with group theory. This will allow us to solve (negatively) the age-old problems of trisecting the angle, duplicating the cube, and solving polynomial equations by radicals. At the end of the course we will remark on the problem of squaring the circle.

The topics are foundational and covered in many textbooks – the suggestions here are not meant to be exclusive. Basically, any textbook titled "Algebra", "Abstract Algebra" will cover everything. Examples include [1, 2, 3, 4]. Similarly, any textbook titled "Galois Theory" such as [5, 6] will be sufficient.

Teaching and learning

Significant prerequisites

- Linear algebra at the level of UBC Math 223: vector spaces, linear maps, Gaussian elimination and linear equations, determinants, eigenvalues and eigenvectors.
- Group theory at the level of UBC Math 322.
- Ring theory at the level of UBC Math 323: rings, ideals, quotients, polynomial rings, modular arithmetic.

Learning goals

- Mathematical: the course plan will be posted on the course website.
- Metamathematical: applying machinery to problems, connecting ideas from different fields of mathematics.

Expectations

You can expect from me:

- To come prepared for class: knowing what we want to achieve, and how we will achieve it.
- Responses to your questions and concerns: continuously in class and during my office hours, within reasonable time by e-mail outside class.
- Demanding homework and examinations.
- Clear explanations of what is correct in your work and what is not, and help in improving.

You are expected:

- To come prepared to class, having read relevant material and done problem sets.
  - Working on the problem sets is *absolutely essential* for learning the material. **It is extremely rare for students who skip problem sets to do well on exams.**
- Actively participate in the course: read ahead of class, think about the material, and **ask questions**.
  - Asking questions when you don't understand, or want to learn more, ensures that you get what you want out of the course. Ask me questions in class, by email, and during office hours. Also, ask your colleagues questions outside of class – you will both benefit from the discussion!
- Written work that is readable and communicates your ideas.

Official Policies

General policies

- **Late or missed exams and assignments will not be accepted for credit and will be given a grade of zero**. In exceptional circumstances the missed work will be registered if you finish it and hand it in when you can.
  - Registered homework will not count when the homework average is computed. The weight of a registered midterm will be transferred to the final exam.
  - If you need to miss work / have missed work please let me know as soon as possible. Sometimes this means letting me know well in advance (example: you are scheduled to represent UBC in an athletic competition later in the term), but sometimes after the fact (example: you fall and break your arm, and write to me only after you are ready to resume schoolwork).
  - In common situations (medical / conflicting responsibilities / compassionate grounds) no documentation is required for the **first request** for a concession. Instead, the student must submit the department's *Academic Concession Form*, available at `https://www.math.ubc.ca/Ugrad/ugradForm/Student_Declaration_Academic_Concession_MATH.pdf`.
  - First requests for concessions on other grounds, and subsequent requests for concessions must include documentation.
  - Full details may be found in Senate Policy 135.
- All assertions in your written work require ***proof*** unless the problem states otherwise. Regardless of the problem's operative word ("find", "solve", "establish", "calculate", "determine" ...), you must rigorously justify your answer.

- Written work should be presented carefully, with sufficient detail in complete English sentences. A "correct sequence of formulas" will only merit partial credit. Examples of the expectations may be distributed together with the first problem set.
- I may designate material (e.g. definitions) for self-study, in which case you are responsible for learning this material before it is used in class and in problem sets.

Homework

- There will be up to twelve problem sets posted to the course website, due at the *beginning* of class on the day shown. I will drop the lowest score when calculating the homework grade.
  - You are encouraged to work on solving the problems together. However, each of you must write your solutions independently, in your own words. You may (and should) share your ideas but you may not share your written work.
  - It is possible that only certain problems from a problem set will be selected for grading.
  - Solutions will be posted on the secure (Canvas) website.
- You will be asked to pledge to not post homework questions to online websites or consult such websites for help. This is absolutely essential: the problems are difficult, but are chosen for pedagogical reasons. Evading the difficulty harms the students who do their best without cheating.

Exams

- There will one midterm exam in class.
  - If you need special accommodations when taking written exams, please contact the Centre for Accessibility.
  - If the midterm (or final) exam conflicts with a religious observance, or if you have any other legitimate conflict, please contact me *at least two weeks ahead of time* so we can make appropriate arrangements.
- There will be a final exam during the usual exam period.

Final grade

- The final grade will be calculated as follows:

Problem sets: 40%
Midterm: 30%
Final exam: 30%

UBC boilerplate

UBC provides resources to support student learning and to maintain healthy lifestyles but recognizes that sometimes crises arise and so there are additional resources to access including those for survivors of sexual violence. UBC values respect for the person and ideas of all members of the academic community. Harassment and discrimination are not tolerated nor is suppression of academic freedom. UBC provides appropriate accommodation for students with disabilities and for religious, spiritual and cultural observances. UBC values academic honesty and students are expected to acknowledge the ideas generated by others and to uphold the highest academic standards in all of their actions. Details of the policies and how to access support are available here.

References

[1] Michael Artin. *Algebra*. Prentice Hall Inc., Englewood Cliffs, NJ, 1991.

[2] David S. Dummit and Richard M. Foote. *Abstract algebra*. John Wiley & Sons Inc., Hoboken, NJ, third edition, 2004.

[3] Nathan Jacobson. *Basic algebra. I*. W. H. Freeman and Company, New York, second edition, 1985.

[4] Serge Lang. *Algebra*, volume 211 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, third edition, 2002.

[5] James Milne. Field and galois theory. Course notes available at `http://www.jmilne.org/math/CourseNotes/math594f.html`.

[6] Ian Stewart. *Galois Theory*. Chapman & Hall/CRC Mathematics. Chapman & Hall/CRC, Boca Raton, FL, third edition, 2004.