Math 312, Winter Term 2021
Updated Midterm 1 Information

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Material

The material for the exam consists of the material covered in the lectures up to and including Monday, January 25th, as well as Problem Sets 1 and 2. Here are some headings for the topics we covered (this is not comprehensive)

- Foundations of the natural numbers: well-ordering, proof by induction.
- Foundations of the integers: divisibility and division with remainder.
- The integers: GCD and LCM, Euclid's Algorithm and Bezout's Theorem, primes and unique factorization, irrational numbers.

Procedure

1. The exam is in-class; students must turn their cameras on during the exam.
2. The exam is closed-book – no materials, notes, or calculators may be used.
   - This is intended to make the exam easier, by allowing questions like "do this calculation" or "state this theorem".
   - An open-book exam with calculators will have to consist primarily of proof problems.
3. Each student will be assigned a random identifier through the Canvas gradebook, and use it to access a PDF from the course website. Test documents will be made available two days before the exam; actual exam papers will be available at a similar URL when the exam begins. If your exam code is 123,456,789,012 then you will find the test paper at `https://www.math.ubc.ca/~lior/teaching/2021/312_W21/Exams/Test0/123456789012.pdf` (note no commas) and the actual test paper will be at the subdirectory Test1 instead: `https://www.math.ubc.ca/~lior/teaching/2021/312_W21/Exams/Test1/123456789012.pdf` Future tests will be at directories Test2, Test3 and so on with the same individualized filename.
4. The exam will last 45 minutes. At the end of the exam you will have 10 minutes to scan and upload your exams to a Canvas assignment.
5. If you have questions during the exam don't hesitate to ask the instructor!

Structure

The exam will consist of several problems. Problems can be calculational (only the steps of the calculation are required), theoretical (prove that something holds) or factual (state a Definition, Theorem, etc). The intention is to check that the basic tools are at your fingertips. Generally, earlier problems are easier than latter problems; the number of points a problem is worth should not be used as an indication of difficulty. At least one problem will be taken directly from the homework.

Two sample problems

For sample problems check out the past final exams posted at `http://www.math.ubc.ca/Ugrad/pastExams/index.shtml#312`, and the practic problems by Freitas–Gherga. Here are two more problems illustrating the kind of questions we can have:

1. (Unique factorization)
   (a) [calculational] Write 148 as a product of prime numbers.
   (b) [factual] State the Theorem on unique factorization of natural numbers.
   (c) [factual] Prove that every natural number can be written as a product of primes..
2. [problem] Prove by induction that $a_n = \frac{n(n+1)}{2}$ is an integer for all $n \geq 0$.

Sample solutions

1. (Unique factorization)

   (a) $148 = 2 \cdot 74 = 2^2 \cdot 37$.

   (b) Every positive integer can be written as a product of primes up, uniquely to reordering the factors [Or: Every positive integer can be uniquely represented by a product $\prod_p p^{e_p}$ over all primes p, where $e_p \in \mathbb{Z}_{\geq 0}$ and all but finitely many are zero).

   (c) Assume that there are natural numbers which cannot be written as a product of primes. Then by the well-ordering principle there is a least such integer which we denote n. Then $n > 1$ (1 is the empty product) and n is not prime (it would be equal to the product containing just itself). n must therefore be composite – assume that $n = ab$ with $1 < a, b < n$. Since both a and b are smaller than n, they can both be written as products of primes. Then n is the product of the two products, a contradiction.

2. For $n = 0$ we have $a_0 = 0$, which is an integer. We also have $a_{n+1} - a_n = \frac{(n+1)(n+2)}{2} - \frac{n(n+1)}{2} = \frac{n+1}{2}[n+2-n] = n+1$ so that $a_{n+1} = a_n + n + 1$. It follows that if a_n is an integer so is a_{n+1}.