Math 101 – WORKSHEET 18
IMPROPER INTEGRALS, WORK

1. Comparison of Integrals

(1) Decide which of the following integrals converge
   (a) (103 Final, 2012) $\int_1^\infty \frac{1+\sin x}{x^2}\,\mathrm{d}x$.

   (b) $\int_1^\infty \frac{3-\cos x}{x}\,\mathrm{d}x$.

   (c) (Bell curve) $\int_{-\infty}^{+\infty} e^{-x^2}\,\mathrm{d}x$

   (d) $\int_0^1 \frac{\mathrm{d}x}{\sqrt{x}+\sin x}$

   (e) (hard) $\int_0^1 \frac{\mathrm{d}x}{x^2+x^3}$

   (f) (hard) $\int_0^\infty \frac{x^{1000}}{e^x}\,\mathrm{d}x$

Date: 15/2/2017, Worksheet by Lior Silberman. This instructional material is excluded from the terms of UBC Policy 81.

2. Work

(1) (Preliminary) A worker carries a 20kg bucket to the top of a 10m tall building. Half way up the worker picks up a second 20kg bucket. Calculate the work done by the worker in the first half and second half of the carry and hence the total work done.

(2) When a spring is displaced $x$cm from its equillibrium position it exerts a force of $5x$ Newtons (i.e the force is $F(x) = 5x$). Find the work required to stretch the spring from a displacement of 20cm to a displacement of 60cm.

(3) According to Newton's universal law of gravitation, the force between a planet of mass M and a probe of mass m is $F = \frac{GMm}{r^2}$ where r is the distances between them and $G \approx 6.67 \cdot 10^{-11}\text{m}^3\text{kg}^{-1}\text{s}$ is the gravitational constant. Find the work required to lanuch a probe from the surface of the planet (radius R) all the way to infinity.

(4) In the Morse model for a diatomic molecule (e.g. H_2, O_2 etc), when the two atoms are separated by distance x, the force between them is

$$F(x) = 2E\left(1 - e^{-(x-r)}\right)e^{-(x-r)}$$

where r is the separation between the atoms at the equillibrium position and E is a parameter. Find the work required to dissociate the molecule, by taking an atom all the way from separation r to ∞.